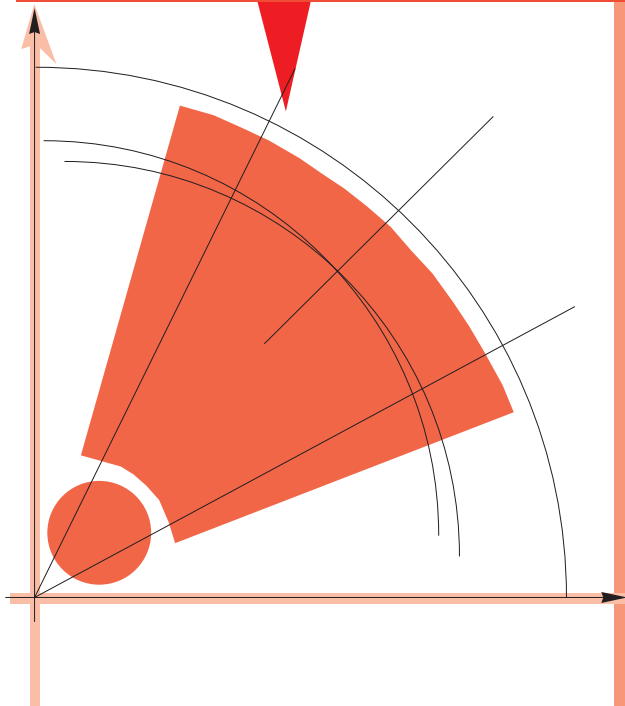


INTERNATIONAL BACCALAUREATE

ANSWERS

**MATHEMATICS
HIGHER LEVEL
(CORE)**



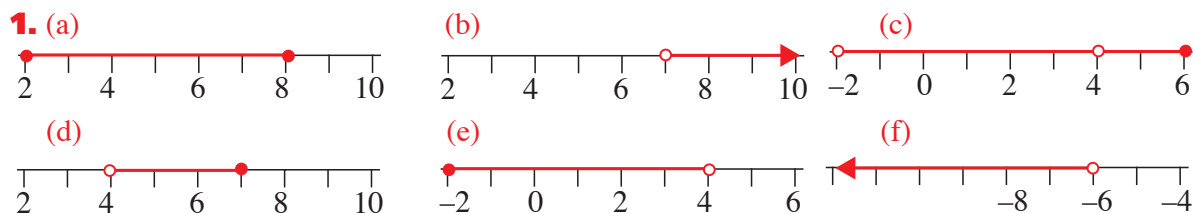
**3RD EDITION
3rd imprint**

Series editor: **Fabio Cirrito**

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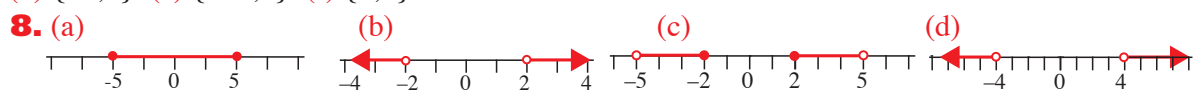
**Nigel Buckle
Iain Dunbar**

EXERCISE 2.1



2. (a) $[-2, 7]$ (b) $[9, \infty[$ (c) $]0, 5]$ (d) $]-\infty, 0]$ (e) $]-4, 8[$ (f) $]-\infty, -1[\cup]2, \infty[$ 3. (a) $5\sqrt{5}$ (b) $-\sqrt{3}$
 (c) $\sqrt{3}$ 4. (a) 4 (b) $4 + \sqrt{6}$ (c) $6\sqrt{2}$ (d) $31 + 12\sqrt{3}$ 5. (a) $2 - \sqrt{3}$ (b) $\sqrt{7} + 2$ (c) $2\sqrt{3} + \sqrt{15}$
 (d) $-2 - \sqrt{3} - 4\sqrt{5} - 2\sqrt{15}$ (e) $\frac{3 + \sqrt{6} + \sqrt{10} + \sqrt{15}}{-2}$ (f) $3\sqrt{6} + 2\sqrt{15}$ 6. (a) i. $\frac{3\sqrt{5} + \sqrt{3}}{2}$

- ii. $10 + \frac{3\sqrt{15}}{2}$ (b) i. $\frac{14\sqrt{3} + 48}{13}$ ii. $\frac{1344\sqrt{3} + 3230}{169}$ 7. (a) $\{\pm 3\}$ (b) $\{\pm 10\}$ (c) \emptyset
 (d) $\{-4, 2\}$ (e) $\{-12, 8\}$ (f) $\{0, 4\}$



9. (a) $]1, \infty[$ (b) $]4, \infty[$ (c) $]4, 6[$ 11. (a) $\sqrt{6} + \sqrt{3}$ (b) $2\sqrt{2} - 2$

EXERCISE 2.2.1

1. (a) 4 (b) 3 (c) -6 (d) $-\frac{11}{2}$ (e) $\frac{1}{10}$ (f) $\frac{3}{8}$ 2. (a) $\frac{17}{5}$ (b) $\frac{4}{3}$ (c) $-\frac{3}{4}$ (d) $\frac{4}{3}$ (e) $\frac{35}{2}$ (f) $\frac{92}{41}$
 3. (a) $-\frac{44}{5}$ (b) -39 (c) $-\frac{1}{7}$ (d) -3 (e) 2 (f) 4 4. (a) $2b - 2$ (b) $b + 1 + \frac{b}{a}$ (c) $\frac{ab}{a+b}$
 (d) $a(a+b)$ (e) ab (f) $\frac{ab}{a-b}$ (g) 0 (h) $\frac{a+b}{a^2+b^2}$ (i) $a+b$ 5. (a) -4, 4 (b) $-\frac{9}{5}, 3$ (c) -6, 18
 (d) $-\frac{11}{2}, \frac{17}{2}$ (e) $-\frac{7}{10}, \frac{1}{10}$ (f) $-\frac{5}{8}, \frac{3}{8}$ (g) $-\frac{7}{5}, \frac{17}{5}$ (h) $\frac{4}{3}, \frac{20}{3}$ (i) -3, 0 (j) $\frac{a-b}{2}, \frac{b-a}{2}, a \geq b$
 (k) $\pm b(a-b), a \geq b$ (l) $-\frac{b}{a}, \frac{2b}{a}, b \geq 0$ 6. (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 0 (e) $-3, \frac{1}{3}$ (f) $-\frac{1}{4}$
 7. (a) $-\frac{2a}{3}, 0$ (b) $\pm a$ (c) \emptyset (d) 0, 4a 8. (a) $\pm \frac{3}{2}$ (b) $x \geq 1$

EXERCISE 2.2.2

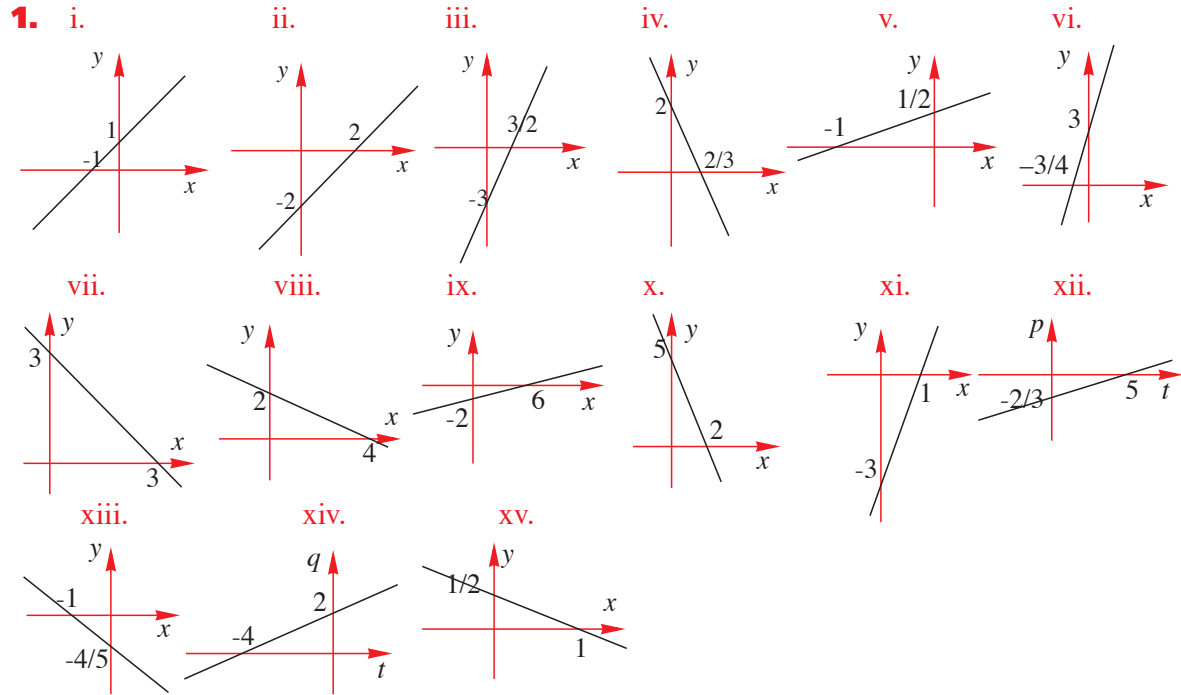
1. (a) $x < -4$ (b) $x \leq -\frac{1}{5}$ (c) $x > 1$ (d) $x \leq -6$ (e) $x > \frac{18}{7}$ (f) $x > \frac{3}{8}$ 2. (a) $x > \frac{52}{11}$ (b) $x \leq 1$
 (c) $x \leq \frac{10}{3}$ 3. (a) $x < 1$ (b) $x < 2 - a$ (c) $x > \frac{2b}{3a}$ (d) $x \geq \frac{2}{(a+1)^2}$ 4. (a) $-2 \leq x \leq 1$
 (b) $-2 \leq x \leq 3$ (c) $-\frac{3}{2} \leq x \leq \frac{5}{2}$ (d) $x = -\frac{1}{2}$ (e) $-7 \leq x \leq 9$ (f) $-5 \leq x \leq 3$ (g) $-4 \leq x \leq 16$
 (h) $-28 \leq x \leq 44$ (i) $-\frac{5}{12} \leq x \leq \frac{1}{12}$ 5. (a) $x < -\frac{3}{2} \cup x > \frac{5}{2}$ (b) $x < \frac{3}{2} \cup x > \frac{7}{2}$ (c) $x \leq -12 \cup x \geq 16$
 (d) $x \leq -24 \cup x \geq 6$ (e) $x < \frac{3}{4} \cup x > \frac{9}{4}$ (f) $-6 < x < 14$ (g) $x < -28 \cup x > 44$

(h) $x < -\frac{5}{12} \cup x > \frac{1}{12}$ (i) $x \leq -4 \cup x \geq 16$ **6.** $p < 3$ **7.** (a) $-\frac{2}{3} < x < 2$ (b) $-3 \leq x \leq 1$

(c) $0 < x < 2$ **8.** (a) $\frac{a}{1+a} < x < \frac{a}{1-a}$ (b) $\frac{-1}{1+a} < x < \frac{1}{1-a}$ (c) $]-\infty, \frac{-a^2}{a+1}] \cup [\frac{a^2}{a-1}, \infty[$

9. (a) $-\frac{4}{3} < x < \frac{4}{3}$ (b) $-\frac{3}{2} < x < \frac{3}{4}$

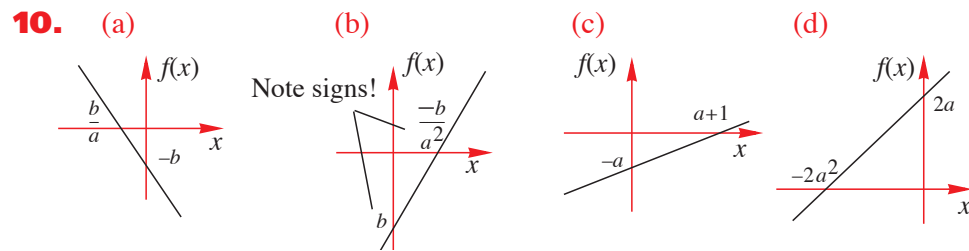
EXERCISE 2.3.1



2. (a) 2 (b) 3 (c) $\frac{5}{3}$ **3.** (a) $y = 2x - 1$ (b) $y = 3x + 9$ (c) $y = -x - 1$ **4.** (a) $-\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{2}$

(d) $-\frac{4}{5}$ **5.** $y = 2x$ **6.** $y = -x + 1$ **7.** $y = \frac{x+2}{2}$ **8.** 2 **9.** (a) $y = \frac{5}{2}x$ (b) $y = -\frac{3}{2}x + 3$

(c) $y = \frac{5}{6}x - \frac{1}{2}$ (d) $y = -2x + 1$



EXERCISE 2.3.2

1. (i) $x = 1, y = 2$ (ii) $x = 3, y = 5$ (iii) $x = -1, y = 2$ (iv) $x = 0, y = 1$ (v) $x = -2, y = -3$

(vi) $x = -5, y = 1$ **2.** (i) $x = \frac{13}{11}, y = \frac{17}{11}$ (ii) $x = \frac{9}{14}, y = \frac{3}{14}$ (iii) $x = 0, y = 0$

(iv) $x = \frac{4}{17}, y = -\frac{22}{17}$ (v) $x = -\frac{16}{7}, y = \frac{78}{7}$ (vi) $x = \frac{5}{42}, y = -\frac{3}{28}$ **3.** (i) -3 (ii) -5 (iii) -1.5

4. (i) $m = 2, a = 8$ (ii) $m = 10, a = 24$ (iii) $m = -6, a = 9$.

5. (a) $x = 1, y = a - b$ (b) $x = -1, y = a + b$ (c) $x = \frac{1}{a}, y = 0$ (d) $x = b, y = 0$

(e) $x = \frac{a-b}{a+b}, y = \frac{a-b}{a+b}$ (f) $x = a, y = b - a^2$

EXERCISE 2.3.3

1. (a) $x = 4, y = -5, z = 1$ (b) $x = 0, y = 4, z = -2$ (c) $x = 10, y = -7, z = 2$
 (d) $x = 1, y = 2, z = -2$ (e) \emptyset (f) $x = 2t - 1, y = t, z = t$ (g) $x = 2, y = -1, z = 0$ (h) \emptyset

EXERCISE 2.4.1

1. (a) -5 (b) $4, 6$ (c) $-3, 0$ (d) $1, 3$ (e) $-6, 3$ (f) $-2, \frac{5}{3}$ (g) 2 (h) $-3, 6$ (i) $-6, 1$ (j) $0, \frac{3}{2}$

2. (a) -1 (b) $-7, 5$ (c) $-\frac{2}{5}, 3$ (d) $-2, 1$ (e) $-3, 1$ (f) $4, 5$

3. (a) $-1 \pm \sqrt{6}$ (b) $3 \pm \sqrt{5}$ (c) $1 \pm \sqrt{5}$ (d) $\frac{-1 \pm \sqrt{33}}{8}$ (e) $\frac{9 \pm \sqrt{73}}{4}$ (f) $\frac{1 \pm \sqrt{85}}{6}$

4. (a) $\frac{3 \pm \sqrt{37}}{2}$ (b) $\frac{5 \pm \sqrt{33}}{2}$ (c) $\frac{3 \pm \sqrt{33}}{2}$ (d) $\frac{7 \pm \sqrt{57}}{2}$ (e) $\frac{-7 \pm \sqrt{65}}{2}$ (f) $-4, 2$ (g) $-1 \pm 2\sqrt{2}$

(h) $\frac{-5 \pm \sqrt{53}}{2}$ (i) $\frac{3 \pm \sqrt{37}}{2}$ (j) no real solutions (k) $4 \pm \sqrt{7}$ (l) no real solutions (m) $\frac{2 \pm \sqrt{13}}{2}$

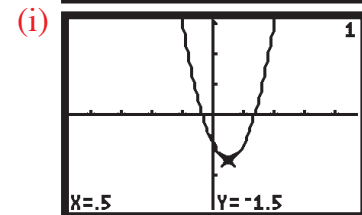
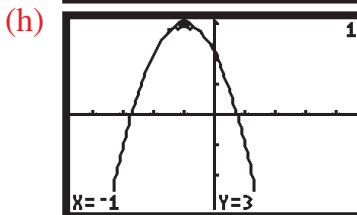
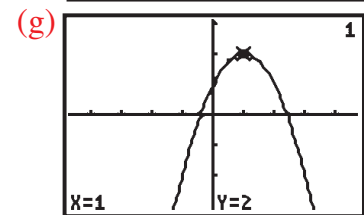
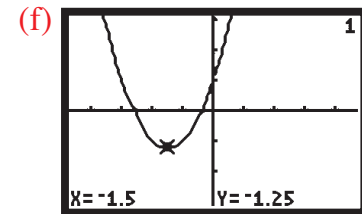
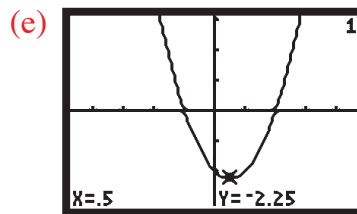
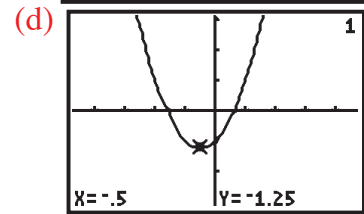
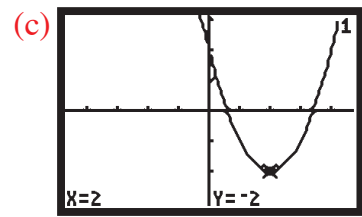
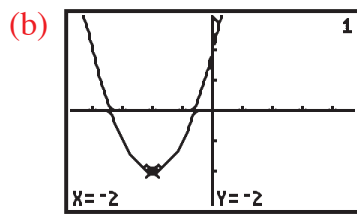
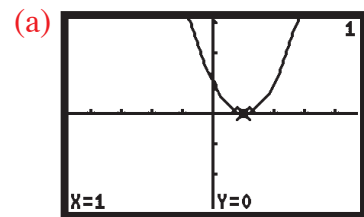
(n) $\frac{3 \pm 2\sqrt{11}}{5}$ (o) $\frac{6 \pm \sqrt{31}}{5}$ (p) $\frac{6 \pm \sqrt{29}}{7}$ 5. (a) $-2 < p < 2$ (b) $p = \pm 2$ (c) $p < -2$ or $p > 2$

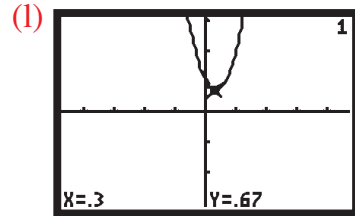
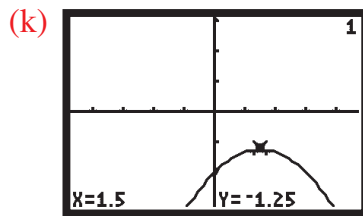
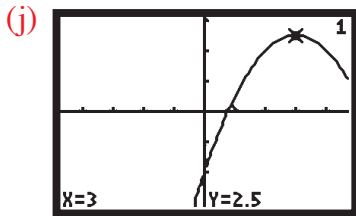
6. (a) $m = 1$ (b) $m < 1$ (c) $m > 1$ 7. (a) $m = \pm 2\sqrt{2}$ (b) $]-\infty, -2\sqrt{2}[\cup]2\sqrt{2}, \infty[$

(c) $]-2\sqrt{2}, 2\sqrt{2}[$ 8. (a) $k = \pm 6\sqrt{2}$ (b) $]-\infty, -6\sqrt{2}[\cup]6\sqrt{2}, \infty[$ (c) $]-6\sqrt{2}, 6\sqrt{2}[$ 10. 4

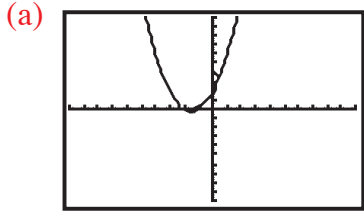
EXERCISE 2.4.2

1. Graphs are shown using the ZOOM4 viewing window:

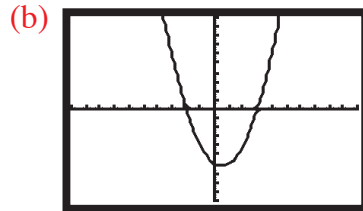




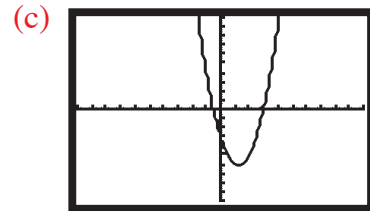
2. Graphs are shown using the ZOOM6 viewing window:



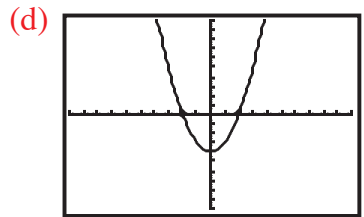
$(-2,0), (-1,0), (0,2)$



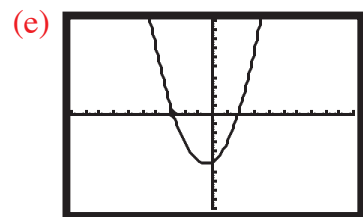
$(-2,0), (3,0), (0,-6)$



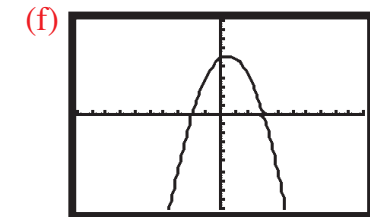
$(-0.5,0), (3,0), (0,-3)$



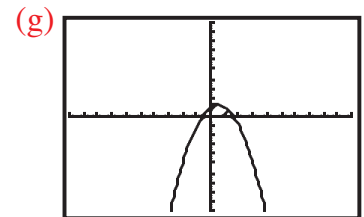
$(-2,0), (2,0), (0,-4)$



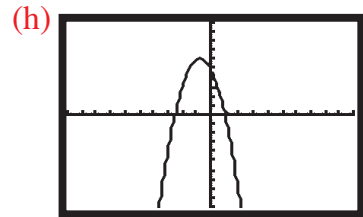
$(-2.79,0), (1.79,0), (0,-5)$



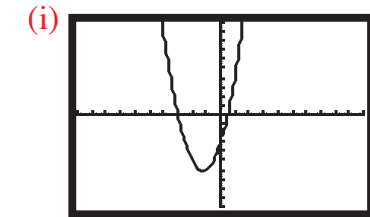
$(-2,0), (3,0), (0,6)$



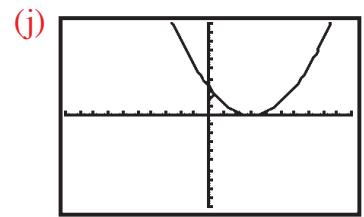
$(-0.62,0), (1.62,0), (0,1)$



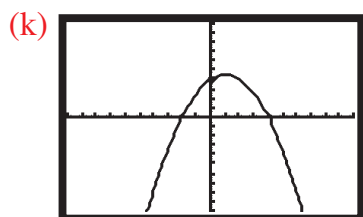
$(-2.5,0), (1,0), (0,5)$



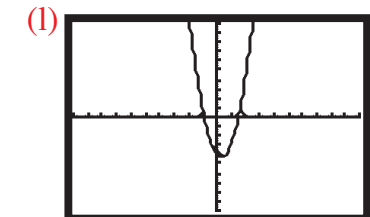
$(-3,0), (0.5,0), (0,-3)$



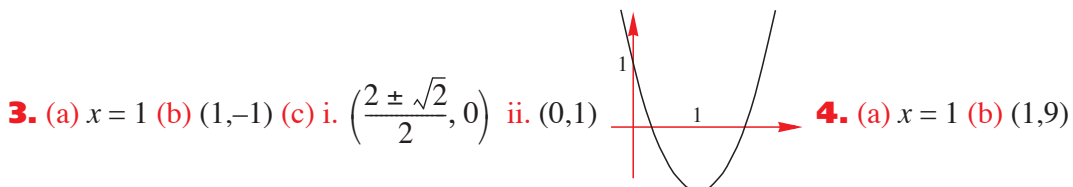
$(3,0), (0,3)$



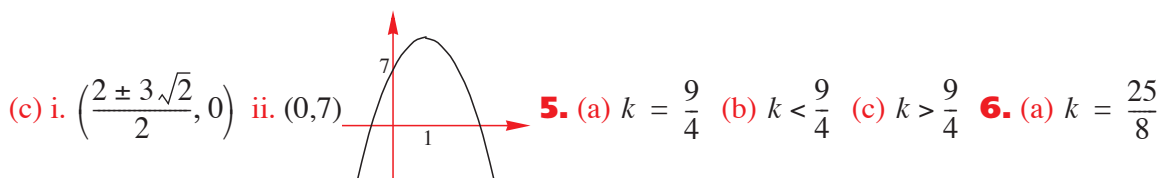
$(-2,0), (4,0), (0,4)$



$(-0.87,0), (1.54,0), (0,-4)$



4. (a) $x = 1$ (b) $(1, 9)$



5. (a) $k = \frac{9}{4}$ (b) $k < \frac{9}{4}$ (c) $k > \frac{9}{4}$ 6. (a) $k = \frac{25}{8}$

(b) $k < \frac{25}{8}$ (c) $k > \frac{25}{8}$ 7. (a) $k = \pm 1$ (b) $-1 < k < 1$ (c) $k < -1 \cup k > 1$

8. (a) $y = \frac{5}{12}(x-2)(x-6)$ (b) $y = -\frac{3}{8}(x+4)^2$ (c) $y = \frac{3}{4}(x-2)^2 + 1$ (d) $y = 3x^2 - 6x + 7$

9. (a) $y = -\frac{2}{5}x(x-6)$ (b) $y = \frac{3}{4}(x-3)^2$ (c) $y = \frac{7}{9}(x+2)^2 + 3$ (d) $y = -\frac{7}{3}x^2 - 2x + \frac{40}{3}$

EXERCISE 2.4.3

1. (a) $]-\infty, -2[\cup]1, \infty[$ (b) $[-3, 2]$ (c) $]-\infty, 0] \cup [4, \infty[$ (d) $]\frac{1}{3}, 3[$ (e) $]-\infty, -1.5] \cup [-1, \infty[$ (f) $]0.75, 2.5[$

2. (a) $]-\infty, -2[\cup]-1, \infty[$ (b) $]-2, 3[$ (c) $]-\infty, -0.5] \cup [3, \infty[$ (d) $[-2, 2]$ (e) $]=\frac{-1-\sqrt{21}}{2}, \frac{-1+\sqrt{21}}{2}[$

(f) $]-\infty, -2] \cup [3, \infty[$ (g) $]=\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}[$ (h) $[-2.5, 1]$ (i) $]-\infty, -3[\cup]0.5, \infty[$ (j) $]1, 3[$ (k) $]-1, 0.5[$ (l) \emptyset

(m) \emptyset (n) $[-1.5, 5]$ (o) $]-\infty, -2[\cup]\frac{1}{3}, \infty[$ 3. (a) $-1 < k < 0$ (b) $-2\sqrt{2} < k < 2\sqrt{2}$ (c) $n \leq -0.5$

4. (a) i. $]-\infty, -1[\cup]2, \infty[$ ii. $[-1, 2]$ (b) i. $]-\infty, 2[\cup]3, \infty[$ ii. $[2, 3]$ (c) i. $]1, 3[$ ii. $]-\infty, 1] \cup [3, \infty[$

(d) i. $]-\frac{2}{3}, 1[$ ii. $]-\infty, -\frac{2}{3}] \cup [1, \infty[$ (e) i. $]-\infty, -2[\cup]2, \infty[$ ii. $[-2, 2]$ (f) i. $]2-\sqrt{3}, 2+\sqrt{3}[$

ii. $]-\infty, 2-\sqrt{3}] \cup [2+\sqrt{3}, \infty[$ 5.  $]0, 1[$ 6. $[-2, 0.5]$

7. (a) i. $]2-\sqrt{6}, 2-\sqrt{2}[\cup]2+\sqrt{2}, 2+\sqrt{6}[$ ii. $]2(1-\sqrt{2}), 2(1+\sqrt{2}) \setminus \{2\}$ iii. $]2(1-\sqrt{3}), 2(1+\sqrt{3})$

(b) i. $]=\frac{5-\sqrt{13}}{2}, \frac{1+\sqrt{13}}{2}[$ ii. all real values 8. (a) $\{x: x < -3\} \cup \{x: x > 2\}$ (b) $\{x: -1 < x < 4\}$

(c) i. $\{x: x < 0.5\}$ ii. $\{x: -2 < x < 0\}$ 9. (a) i. $]0, 1[$ ($k = 1$); $]-1, 0[$ ($k = -1$) ii. \emptyset (b) $k > 1.25$

EXERCISE 2.4.4

1. (a) $(-2, -3)$ $(2, 5)$ (b) $(-2, -1)$ $(1, 2)$ (c) $(-\frac{1}{3}, -2)$, $(2, 5)$ (d) $(-\frac{3}{2}, -\frac{15}{4})$, $(1, 0)$

(e) $(-\frac{9}{2}, -\frac{19}{4})$, $(1, -2)$ (f) $(\frac{3+\sqrt{73}}{4}, \frac{-3-\sqrt{73}}{8})$, $(\frac{3-\sqrt{73}}{4}, \frac{-3+\sqrt{73}}{8})$

(g) $(\frac{1-\sqrt{13}}{2}, 1-\sqrt{13})$, $(\frac{1+\sqrt{13}}{2}, 1+\sqrt{13})$ (h) no real solutions

(i) $(\frac{1-\sqrt{17}}{2}, \frac{5-3\sqrt{17}}{2})$, $(\frac{1+\sqrt{17}}{2}, \frac{5+3\sqrt{17}}{2})$ (j) $(-2, -3)$, $(2, 1)$ (k) no real solutions

2. (a) $(1, 4)$, $(-7, 84)$ (b) $(\frac{4}{3}, -\frac{56}{9})$, $(\frac{3}{4}, -\frac{7}{4})$ (c) $(0, 2)$, $(3, 23)$ (d) $(-a, -a^2)$, $(\frac{a}{2}, \frac{a^2}{2})$ (e) \emptyset

(f) $(2, 8)$ (g) \emptyset (h) $(\frac{1}{2}, \frac{23}{4})$ 3. (a) $\pm 2\sqrt{6}$ (b) $m < -2\sqrt{6}$, $m > 2\sqrt{6}$ (c) $-2\sqrt{6} < m < 2\sqrt{6}$

4. $\sqrt{80}$ 5. 1.75 7. $-\frac{23}{12}$ 9. $(-\infty, -3) \cup (1, \infty)$ 10. 0.5 11. $c = \frac{a}{m}$

13. (a) i. $(1, 3)$, $(-\frac{14}{3}, \frac{196}{3})$ ii. $(-2, 12)$, $(\frac{7}{3}, \frac{49}{3})$ (c) i. A(1, 3), B(-2, 2) ii. 4 sq. units

EXERCISE 3.1

1. (a) $-2x^3 + 6x^2 + x - 7$ (b) $x^4 - x^3 + 12x^2 + 2x - 9$ (c) $6x^5 - 7x^3 + 9x^2 + 2x - 6$
(d) $2x^7 - 2x^6 - x^5 + 8x^4 - 5x^3 - 2x^2 + 7x - 3$ (e) $4x^6 - 4x^4 + 12x^3 + x^2 - 6x + 9$
(f) $9x^3 - 12x^2 - 18x + 13$ 2. $3x + 1 + \frac{2}{x-1}$ 3. $2x^2 - 3x + 11 - \frac{8}{2x-1}$
4. $x^2 - x - 5 + \frac{19-6x}{x^2-2x+3}$ 5. $2x^2 - 7x + 7 - \frac{17}{x+1}$ 6. $x^2 - 1 + \frac{3-x}{x^2+3}$
7. $(3x-1)(\frac{16}{3} - 3x - 2x^2) + \frac{52}{3}$ 8. 1 9. -1 10. -12 11. 5

EXERCISE 3.2

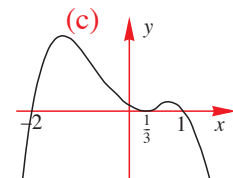
1. $(x-1)(2x-3) - 2$ 2. $(x-3)(3x^2 + 10x + 29) + 90$ 3. $(x+3)(2x^3 - 7x^2 + 19x - 57) + 174$
4. $(2x-1)(x^2 - 2x + 4) + 1$ 5. $(x+2)(2x^3 - 4x^2 + 11x - 23) + 46$
6. $(4-x)(x^3 + 4x^2 + 18x + 72) - 283$

EXERCISE 3.3

1. (a) -6 (b) 15 (c) 8 (d) 1.25 (e) $-\frac{155}{27}$ 2. 6 3. (a) 7 (b) 70 (c) $-21x + 28$ 4. -6 5. (a) 11 (b) 3

EXERCISE 3.4

1. (a) $(x-3)(x-2)(x+5)$ (b) $(x-2)(x^2 + 3x + 5)$ (c) $(x-2)(x-1)(x+2)$
(d) $(x-2)(x+2)(3x+1)$ (e) $(x-3)(x+3)(2x-1)$ (f) $(x-2)(x-1)(x^2 + 3x + 4)$
(g) $(x-2)^2(x+3)$ (h) $(x-2)^2(5x-4)$ (i) $-(x+4)(2x-5)(5x+2)$ (j) $-(x+1)^2(5x-1)$
2. $6x^3 - 4x^2 - 2x + 3$ 3. $-\frac{47}{8}$ 4. 0, $(x+4)(x+1)(x-3)$ 5. $(2x+1)(x+2)^2$
6. $(x-5)(x^2 + x + 2)$ 7. $(x-1)^2(2x-1)(3x+2)$ 8. (a) $-(x-1)(x+2)(3x-1)^2$ (b) 1, -2, $\frac{1}{3}$
9. $a = -1, b = -2$ 11. $a = -2, b = 1$ 13. $a = -9, b = 24, (6x^2 + 9x - 2)$
14. $x^3 - 2x^2 + 8x + 2$ 15. $3x^3 - 5x^2 + 6x + 4$ 16. $a + c = b + d$ 17. -8
18. $a = 1, b = -3, c = 3, d = -1$ 19. $(x-2)(2x+1)(3x+2)$
20. (a) $m = \frac{18}{5}, n = \frac{39}{5}, k = -\frac{78}{5}$ (b) $(x-2), (x+3)$
21. $m = 3, n = -4, k = -12; x^3 + 3x^2 - 4x - 12 = (x-2)(x+2)(x+3)$
22. $k = -2, n = 3$ 24. $a = 3, b = -6; a = -3, b = 6$ 26. $(\alpha^2 + \alpha\beta + \beta^2)x - \alpha\beta(\alpha + \beta)$



EXERCISE 3.5.1

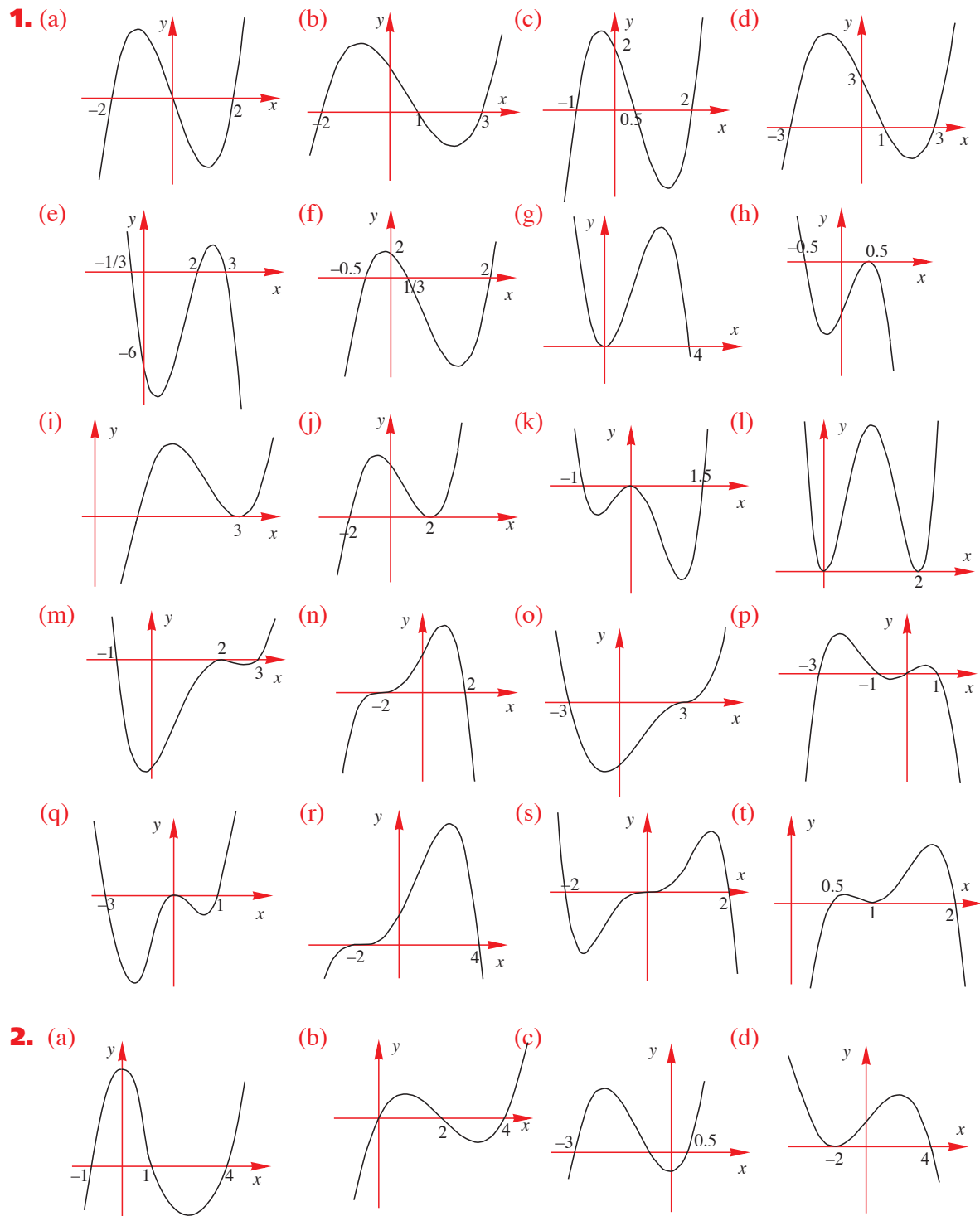
1. (a) -3, -1, 2 (b) $-\frac{1}{2}, 1, 2$ (c) -2, -1, 3 (d) $-\frac{1}{3}, \frac{3}{2}, 4$ (e) $\frac{1}{2}$ (f) -1, 3 (g) -4, 1 (h) -2, $2 \pm \sqrt{10}$
(i) $-\frac{1}{2}, -2 \pm \sqrt{2}$ (j) $\frac{1}{3}, \pm\sqrt{6}$ 2. $\frac{1}{2}, 1, 5$ 3. $-\frac{3}{2}, \frac{1}{2}, 2$ 4. -1, 2, 3 5. -4, 1
6. (a) $(x+2)(x-3)(x+4) = 0$ (b) $(x+1)(x-0.5)(x-2) = 0$ (c) $8x^3 - 16x^2 - 2x + 4 = 0$
7. (a) $\frac{-7 \pm \sqrt{337}}{12}, 1$ (b) -1, 1 (c) -4, -1, 2 (d) $1, \frac{3}{2}, 3$ 8. (a) -1.75, 0.432, 1.32 (b) 3.77
(c) 0.309 (d) -1.68, -0.421, 0.421, 1.68 9. -3, $-\frac{1}{2}, 2$ 10. No other solutions
11. $m = 1, n = -6; x = 6, -1, 2$ 12. 1, 4, 7

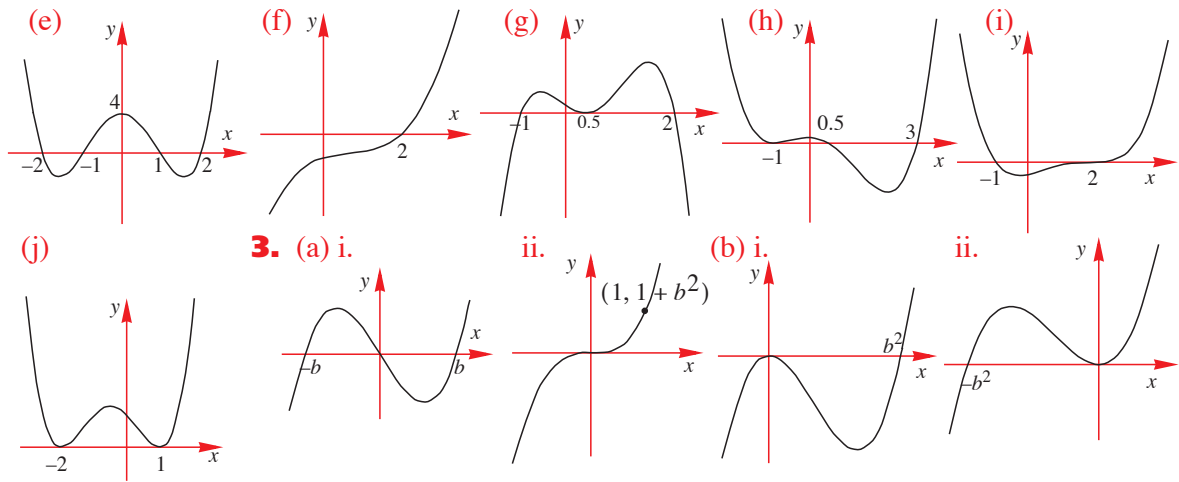
EXERCISE 3.5.2

1. (a) $]-1, 1[\cup]2, \infty[$ (b) $]-\infty, -2] \cup]2, 3[$ (c) $]-3, -2] \cup]2, \infty[$ (d) $]0, \infty[\setminus \{1\}$ (e) $\{-2\} \cup]-0.5, \infty[$
 (f) $]-\infty, -4[\cup]-2, 2[$ (g) $]-\infty, 1[\cup]2, \infty[\setminus \{-1\}$ (h) $]-\infty, 2[\cup \{3\}$

2. (a) $]-3, -1[\cup]2, \infty[$ (b) $]-\infty, -\frac{1}{2}] \cup]1, 2[$ (c) $]-2, -1] \cup]3, \infty[$ (d) $]-\frac{1}{3}, \frac{3}{2}[\cup]4, \infty[$ (e) $[\frac{1}{2}, \infty[$
 (f) $]3, \infty[$ (g) $]-\infty, -4[$ (h) $]-2, 2 - \sqrt{10}] \cup]2 + \sqrt{10}, \infty[$ (i) $]-\infty, -2 - \sqrt{2}] \cup]-2 + \sqrt{2}, -\frac{1}{2}]$
 (j) $]-\infty, -\sqrt{6}[\cup]\frac{1}{3}, \sqrt{6}[$ (k) $]-\infty, 1[$ (l) $]-\infty, \frac{1-\sqrt{13}}{2}[\cup]\frac{1+\sqrt{13}}{2}, \infty[$

EXERCISE 3.6

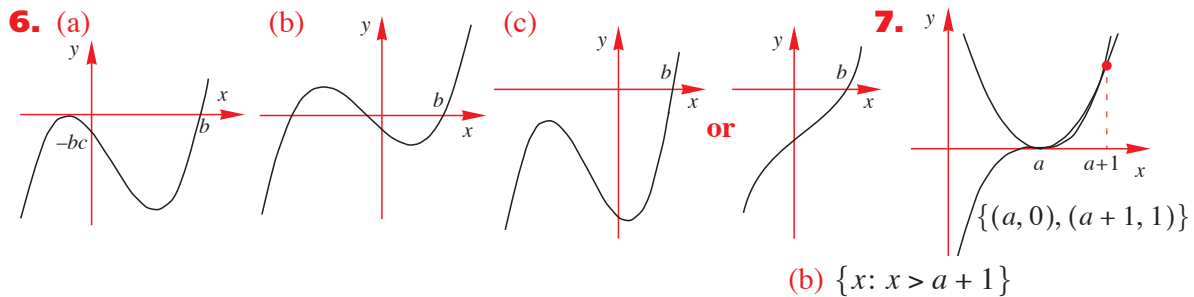




4. (a) $y = -\frac{1}{15}(x+3)(x-1)(x-5)$ (b) $y = \frac{1}{8}(x-2)^2(x+4)$ (c) $y = -\frac{3}{2}x^2(x-3)$

(d) $y = \frac{1}{30}(x+2)^2(75-29x)$ (e) $y = \frac{1}{6}(x+2)(4-3x)(x-3)$ (f) $y = -x^3 - x^2 + 2x + 8$

5. (a) $y = \frac{1}{2}(x+2)(x-2)^3$ (b) $y = \frac{1}{35}x^2(x-3)(x-5)$ (c) $y = -\frac{1}{6}(x+2)^2(x-1)(x-3)$



EXERCISE 4.1.1

1. (a) $b^2 + 2bc + c^2$ (b) $a^3 + 3a^2g + 3ag^2 + g^3$ (c) $1 + 3y + 3y^2 + y^3$

(d) $16 + 32x + 24x^2 + 8x^3 + x^4$ (e) $8 + 24x + 24x^2 + 8x^3$ (f) $8x^3 - 48x^2 + 96x - 64$

(g) $16 + \frac{32}{7}x + \frac{24}{49}x^2 + \frac{8}{343}x^3 + \frac{1}{2401}x^4$ (h) $8x^3 - 60x^2 + 150x - 125$

(i) $27x^3 - 108x^2 + 144x - 64$ (j) $27x^3 - 243x^2 + 729x - 729$ (k) $8x^3 + 72x^2 + 216x + 216$

(l) $b^3 + 9b^2d + 27bd^2 + 27d^3$ (m) $81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$

(n) $x^5 + 15x^4y + 90x^3y^2 + 270x^2y^3 + 405xy^4 + 243y^5$ (o) $\frac{125}{p^3} + \frac{150}{p} + 60p + 8p^3$

(p) $\frac{16}{x^4} - \frac{32}{x} + 24x^2 - 8x^5 + x^8$ (q) $q^5 + \frac{10q^4}{p^3} + \frac{40q^3}{p^6} + \frac{80q^2}{p^9} + \frac{80q}{p^{12}} + \frac{32}{p^{15}}$ (r) $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$

EXERCISE 4.1.2

1. (a) $160x^3$ (b) $21x^5y^2$ (c) $-448x^3$ (d) $-810x^4$ (e) $216p^4$ (f) $-20412p^2q^5$ (g) $-22680p$

2. (a) -1400000 (b) 6000 (c) 540 (d) -240 (e) 81648 (f) 40 **3.** 1.0406 0.0004%

4. i. $64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625$ ii. 19750 iii. 20.6

iv. 0.1% **5.** 19 **6.** $-\frac{63}{8}$ **7.** $\frac{231}{16}$ **8.** $-\frac{130}{27}$ **9.** -20 **10.** $a = \pm 3$ **11.** $n = 5$ **12.** $n = 9$

13. (a) 0 (b) -59 **14.** $a = 3, n = 8$ **15.** $a = \pm 2, b = \pm 1$

EXERCISE 5.1

1. (a) $\text{dom} = \{2, 3, -2\}$, $\text{ran} = \{4, -9, 9\}$ (b) $\text{dom} = \{1, 2, 3, 5, 7, 9\}$, $\text{ran} = \{2, 3, 4, 6, 8, 10\}$
 (c) $\text{dom} = \{0, 1\}$, $\text{ran} = \{1, 2\}$ 2. (a) $]1, \infty[$ (b) $[0, \infty[$ (c) $]9, \infty[$ (d) $]-\infty, 1[$ (e) $[-3, 3]$
 (f) $]-\infty, \infty[$ (g) $]-1, 0[$ (h) $[0, 4[$ (i) $[0, \infty[$ (j) $[1, 5[$ (k) $]0, 4[$ (l) $]-\infty, -1] \cup [1, \infty[$
 3. (a) $r = [-1, \infty[$, $d = [0, 2[$ (b) $r = \{y : y \geq 0\} \setminus \{4\}$, $d = \mathbb{R}$ (c) $r = [0, \infty[\setminus \{3\}$, $d = [-4, \infty[\setminus \{0\}$
 (d) $r = [-2, 0[$, $d = [-1, 2[$ (e) $r =]-\infty, \infty[$, $d =]-\infty, -3] \cup [3, \infty[$ (f) $r = [-4, 4]$, $d = [0, 8]$
 4. (a) one to many (b) many to one (c) many to one (d) one to one (e) many to many
 (f) one to one 5. (a) $\mathbb{R} \setminus \{-2\}$ (b) $]-\infty, 9[$ (c) $[-4, 4]$ (d) $]-\infty, -2] \cup [2, \infty[$ (e) $\mathbb{R} \setminus \{0\}$ (f) \mathbb{R}
 (g) $\mathbb{R} \setminus \{-1\}$ (h) $[-a, \infty[$ (i) $[0, \infty[\setminus \{a^2\}$ (j) $]-\infty, -a] \cup [a, \infty[$ (k) \mathbb{R} (l) $\mathbb{R} \setminus \{-a^{-1}\}$
 6. (a) $]-\infty, -a[$ (b) $]0, ab[$ (c) $]-\infty, \frac{1}{4}a^3]$ (d) $[\frac{1}{4}a^3, \infty[$ (e) $\mathbb{R} \setminus \{a\}$ (f) $]-\infty, a[$ (g) $[-a, \infty[$ (h) $]-\infty, 0[$

EXERCISE 5.2

Graphs with graphics calculator output have standard viewing window unless otherwise stated.

1. (a) 3, 5 (b) i. $2(x+a) + 3$ ii. $2a$ (c) 3 2. (a) $0, \frac{10}{11}$ (b) $-\frac{5}{4}$ (c) $[0, \frac{10}{11}]$

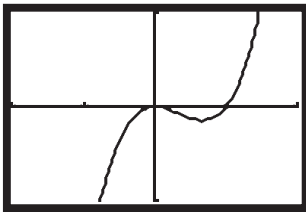
3. (a) $-\frac{1}{2}x^2 - x + \frac{3}{2}$, $-\frac{1}{2}x^2 + x + \frac{3}{2}$ (b) $\pm\sqrt{2}$ (c) no solution

4. (a) $x = 0, 1$

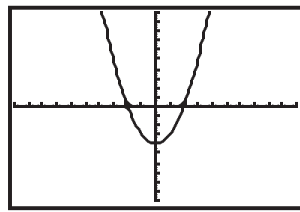
5. (a) i.

ii.

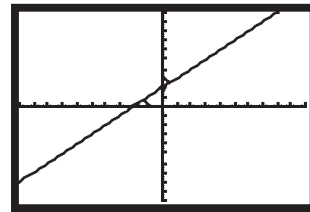
(b)



Window $[-2, 2]$, $[-1, 1]$
 Range: $[-12, 4]$

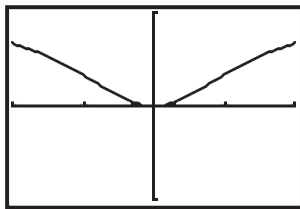


(b) i. $\{2\sqrt{2}, -2\sqrt{2}\}$ ii. $\{3, -2\}$



6. (b), (c), (d), (e) 8. (a), (d), (e), (f) 10. (a) $\{y : y > 1\} \cup \{y : y \leq -1.25\}$ (b) 10 11. (b) 1

9. (a)



Window $[-2, 2]$, $[-1, 1]$

(b) $[0, 1[$

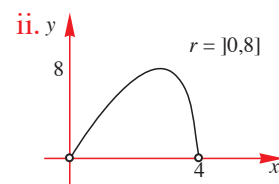
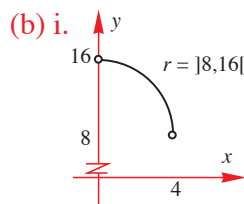
12. (a) only - it is the only one with identical rules &

domains.

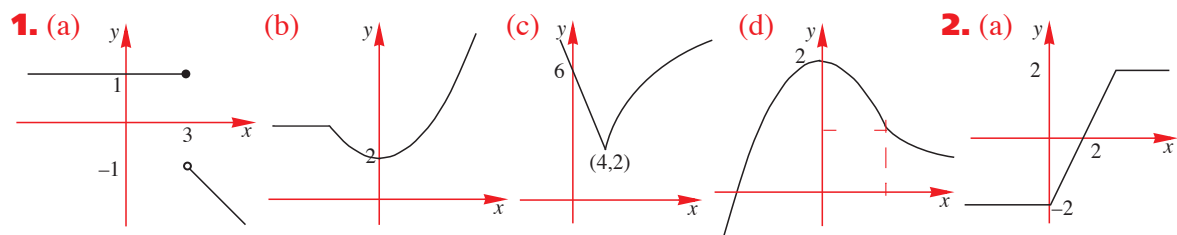
13. (a) $[-3, \infty[$ (b) $[-3, 0]$ (c) $[3, \infty[$ (d) $[1.5, 3[\cup]3, \infty[$

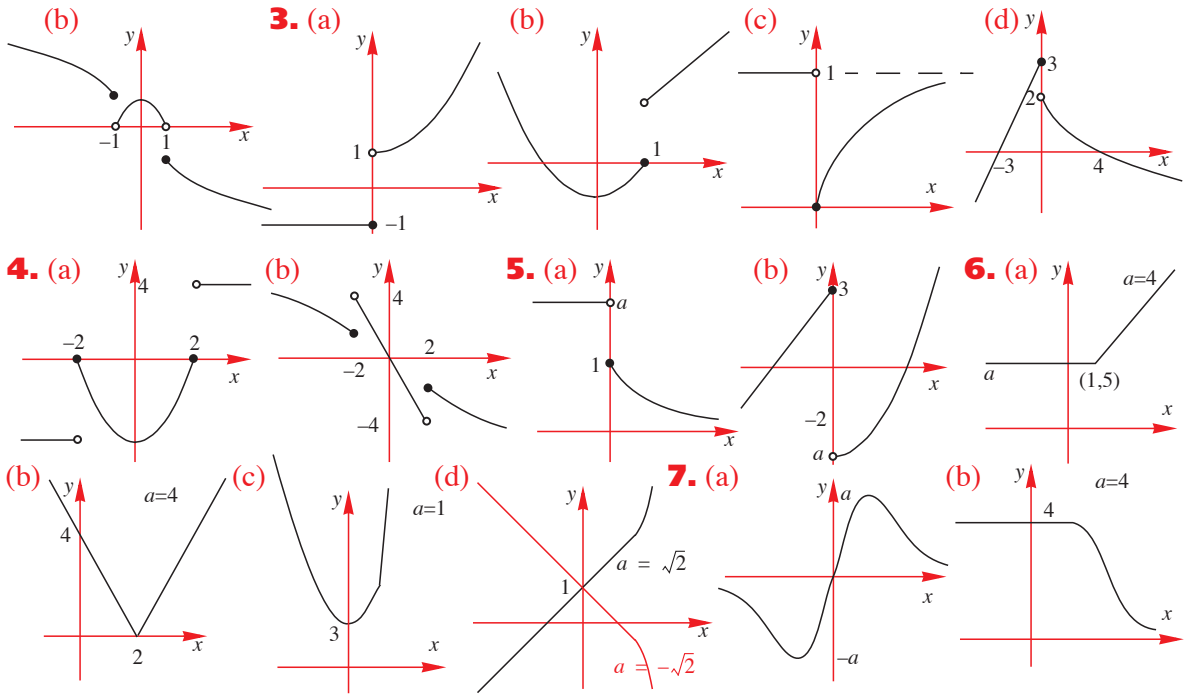
14. (a) i. $p(x) = 8 + 2\sqrt{16 - x^2}$, $0 < x < 4$

ii. $A(x) = x\sqrt{16 - x^2}$, $0 < x < 4$

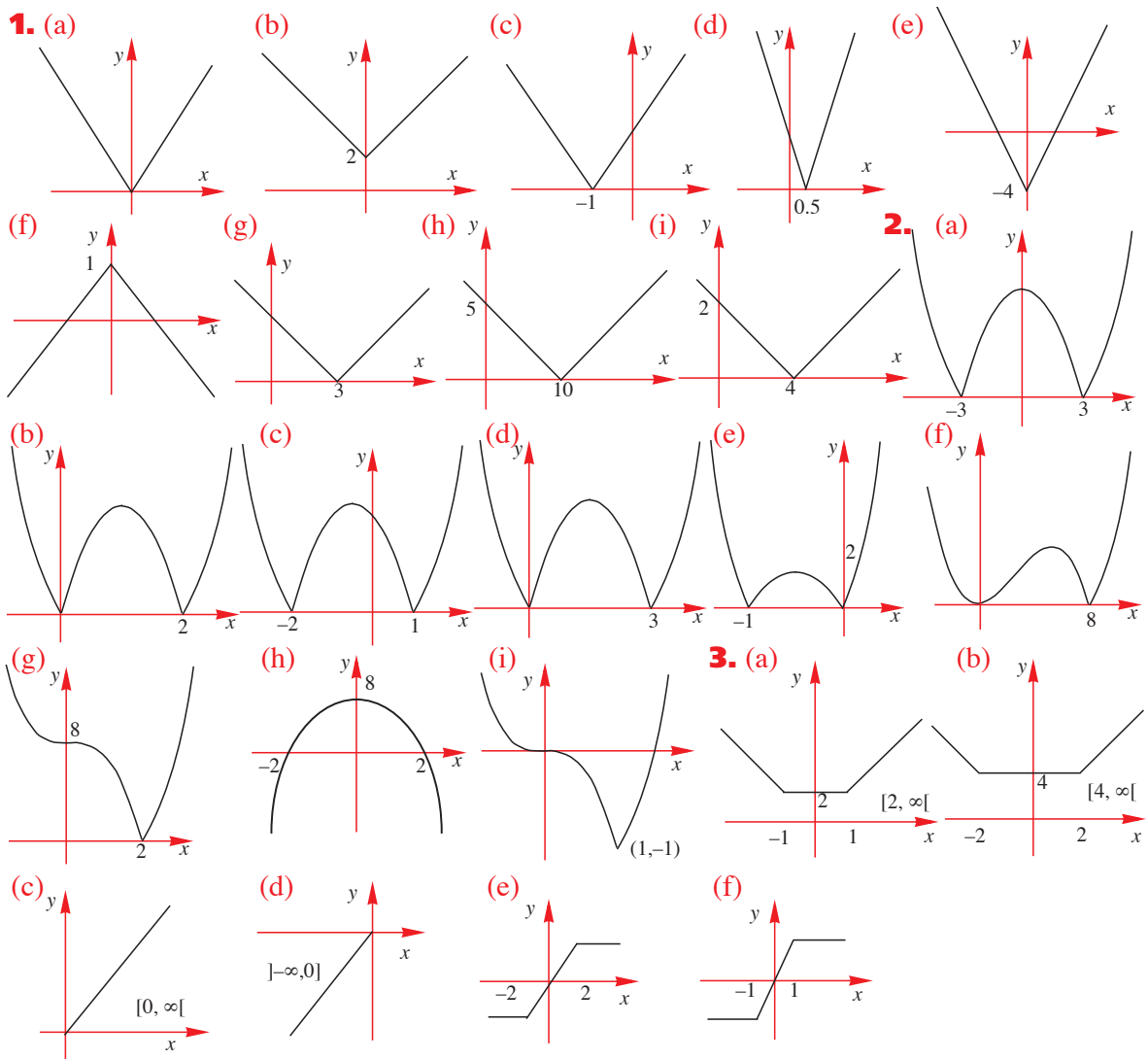


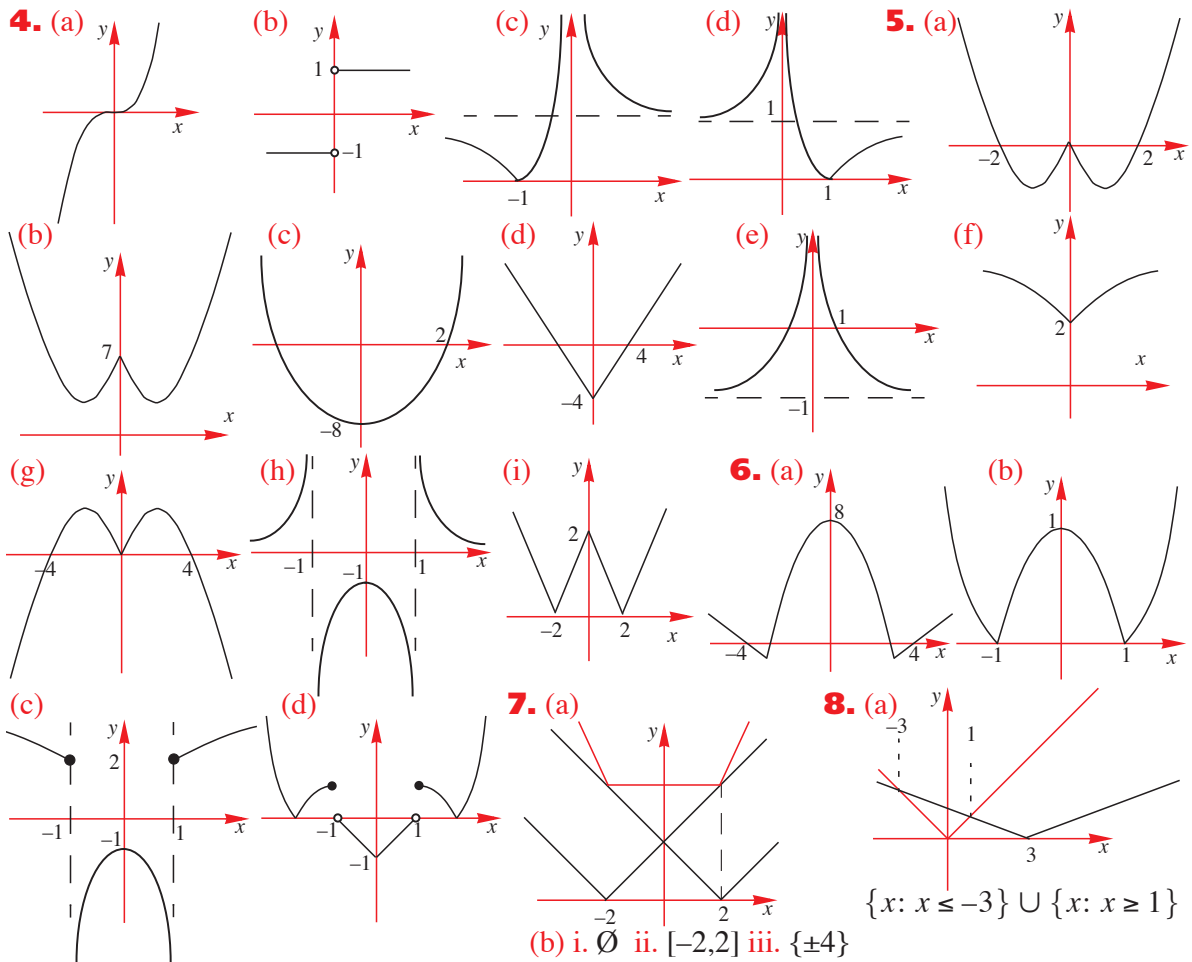
EXERCISE 5.3.1



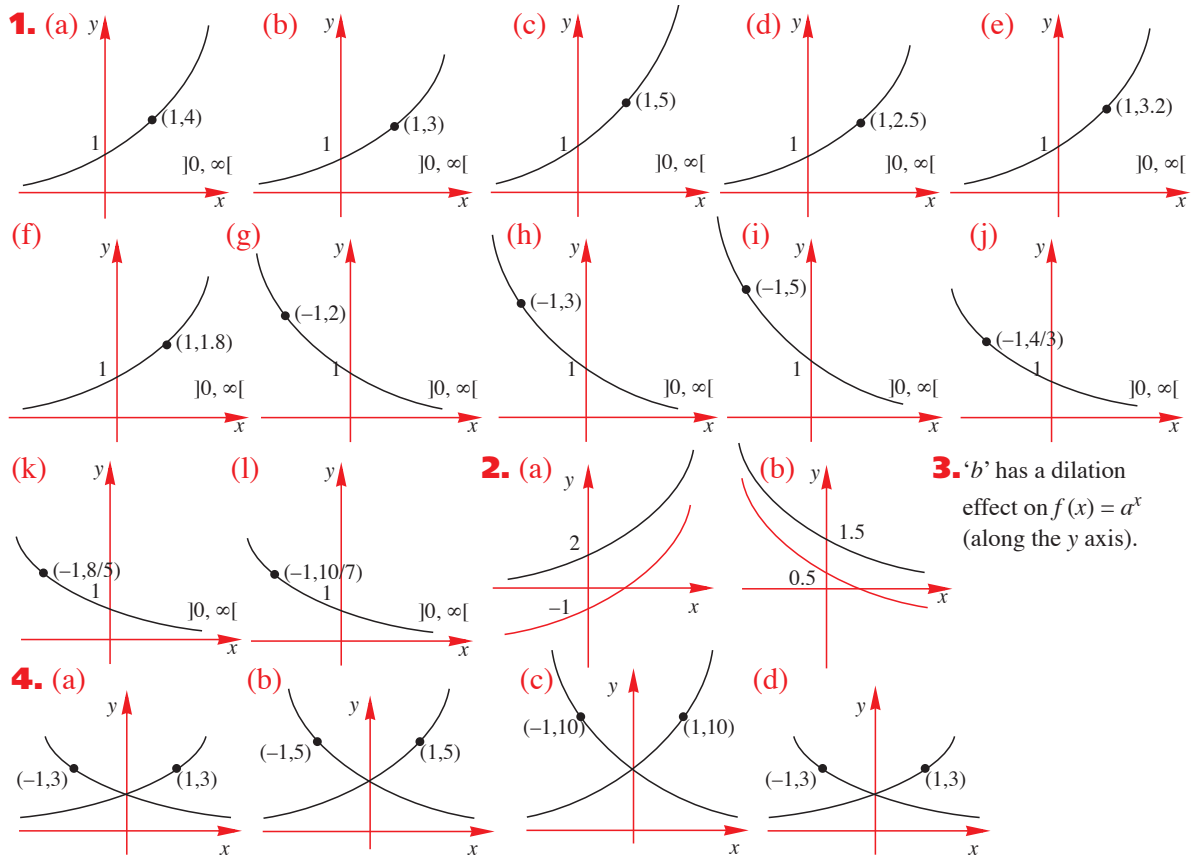


EXERCISE 5.3.2

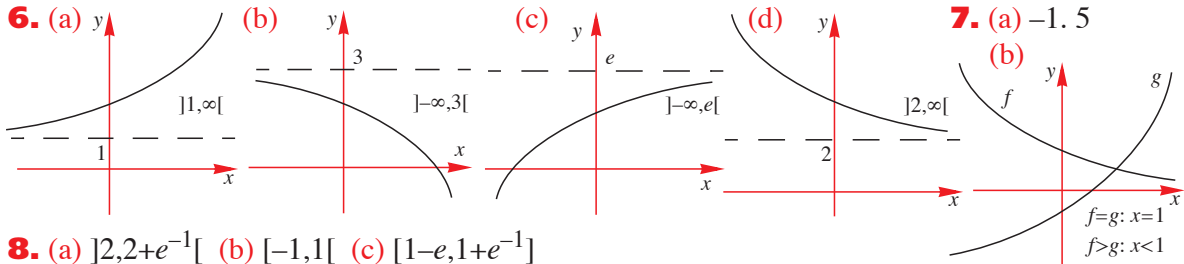




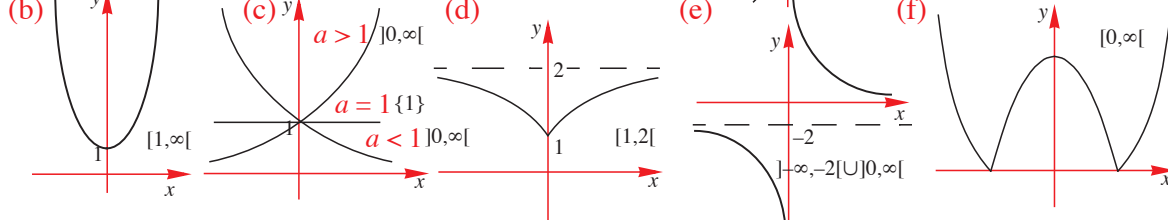
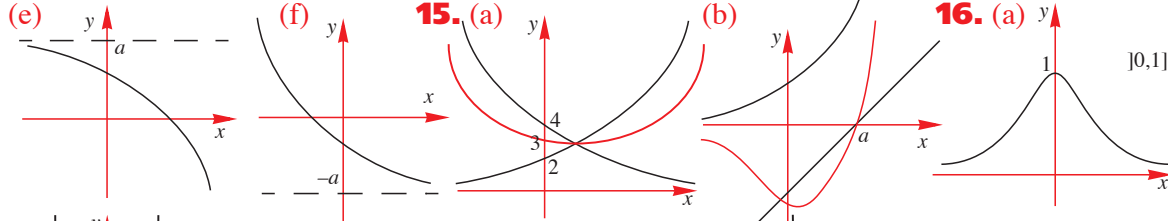
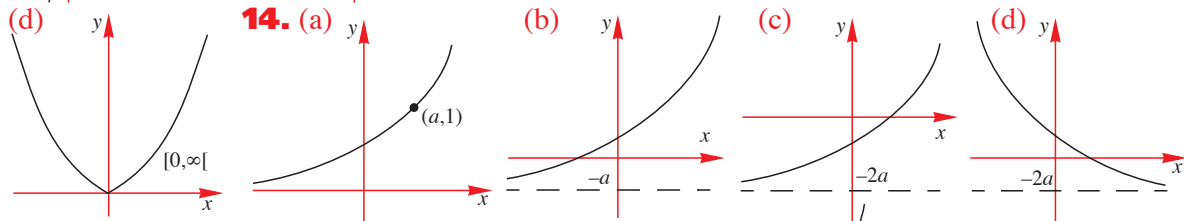
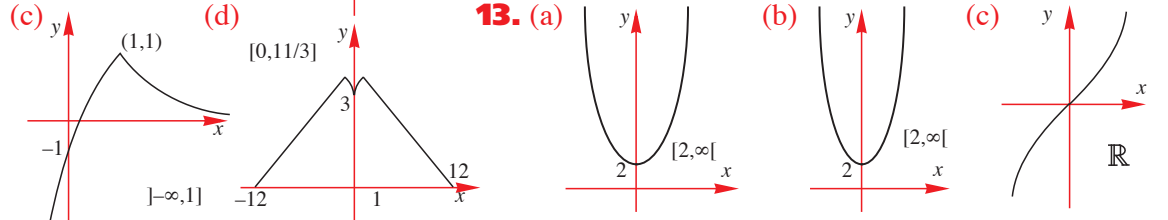
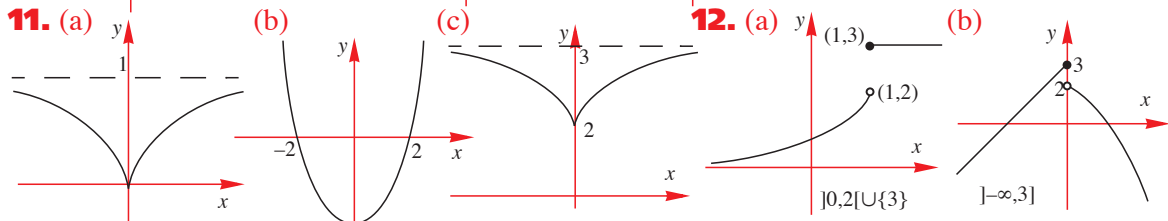
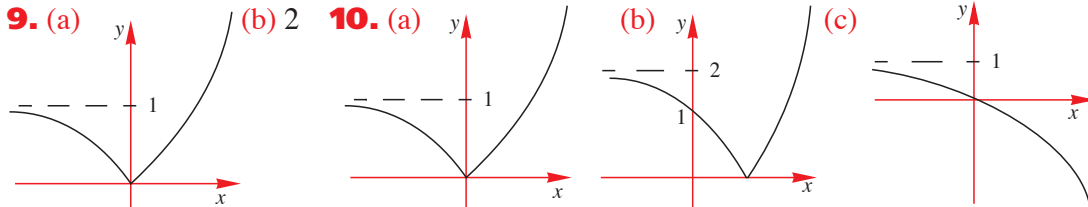
EXERCISE 5.3.3



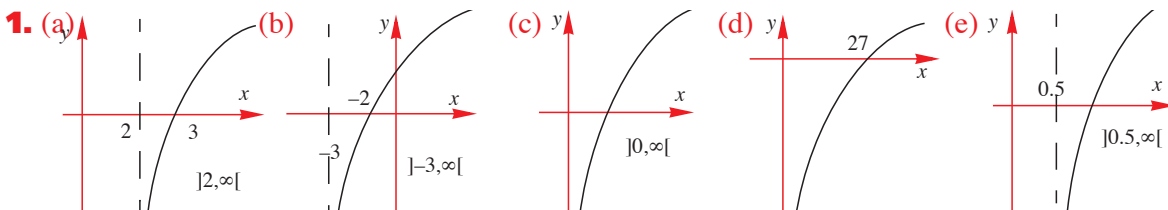
5. (a) $[1, 16]$ (b) $[3, 27]$ (c) $[0.25, 16]$ (d) $[0.5, 4]$ (e) $[0.125, 0.25]$ (f) $[0.1, 10]$

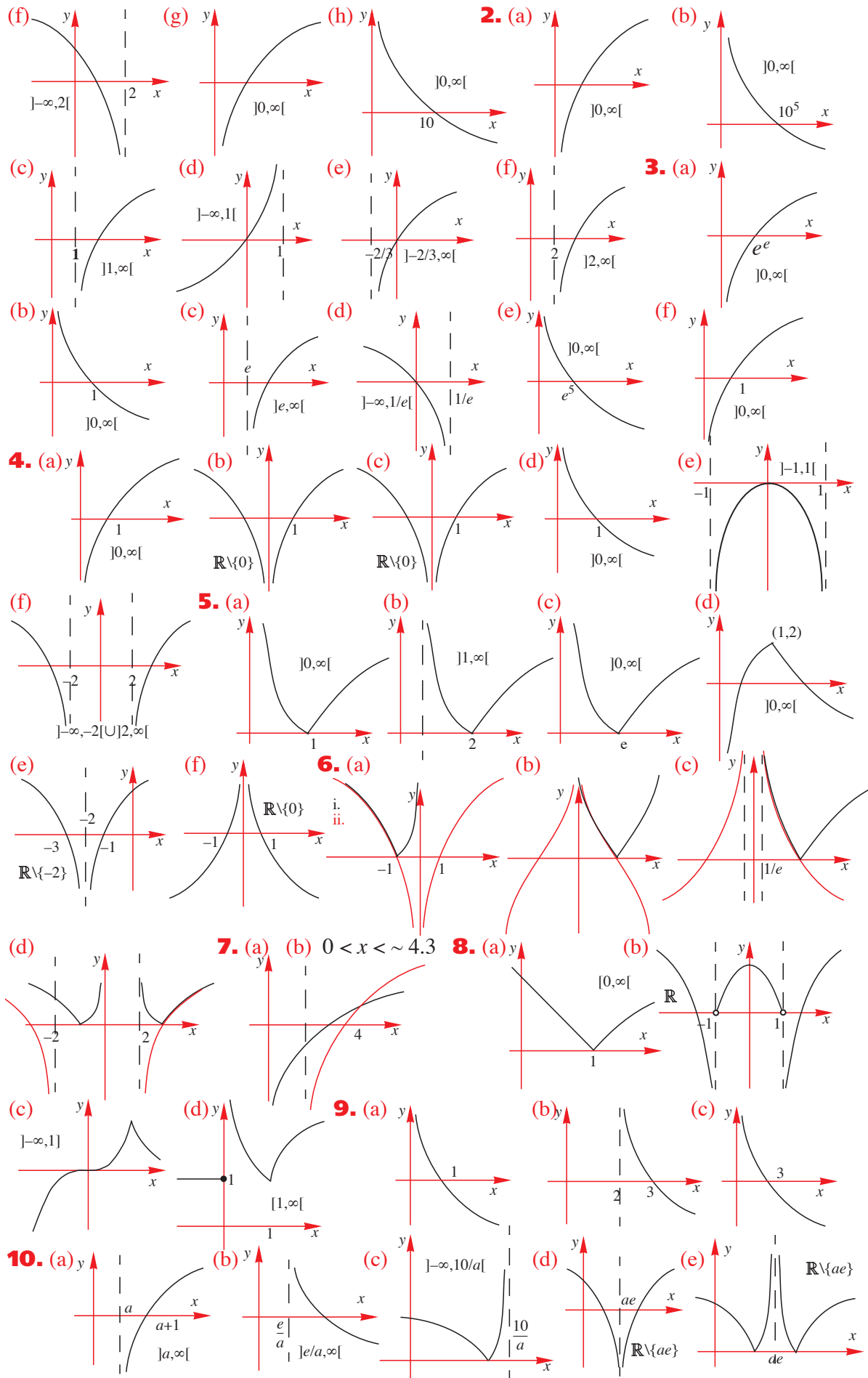


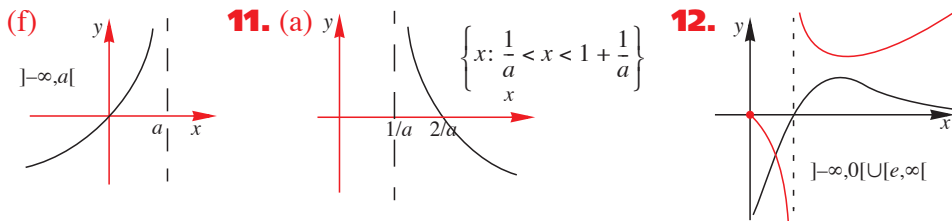
8. (a) $]2, 2+e^{-1}[$ (b) $[-1, 1[$ (c) $[1-e, 1+e^{-1}]$



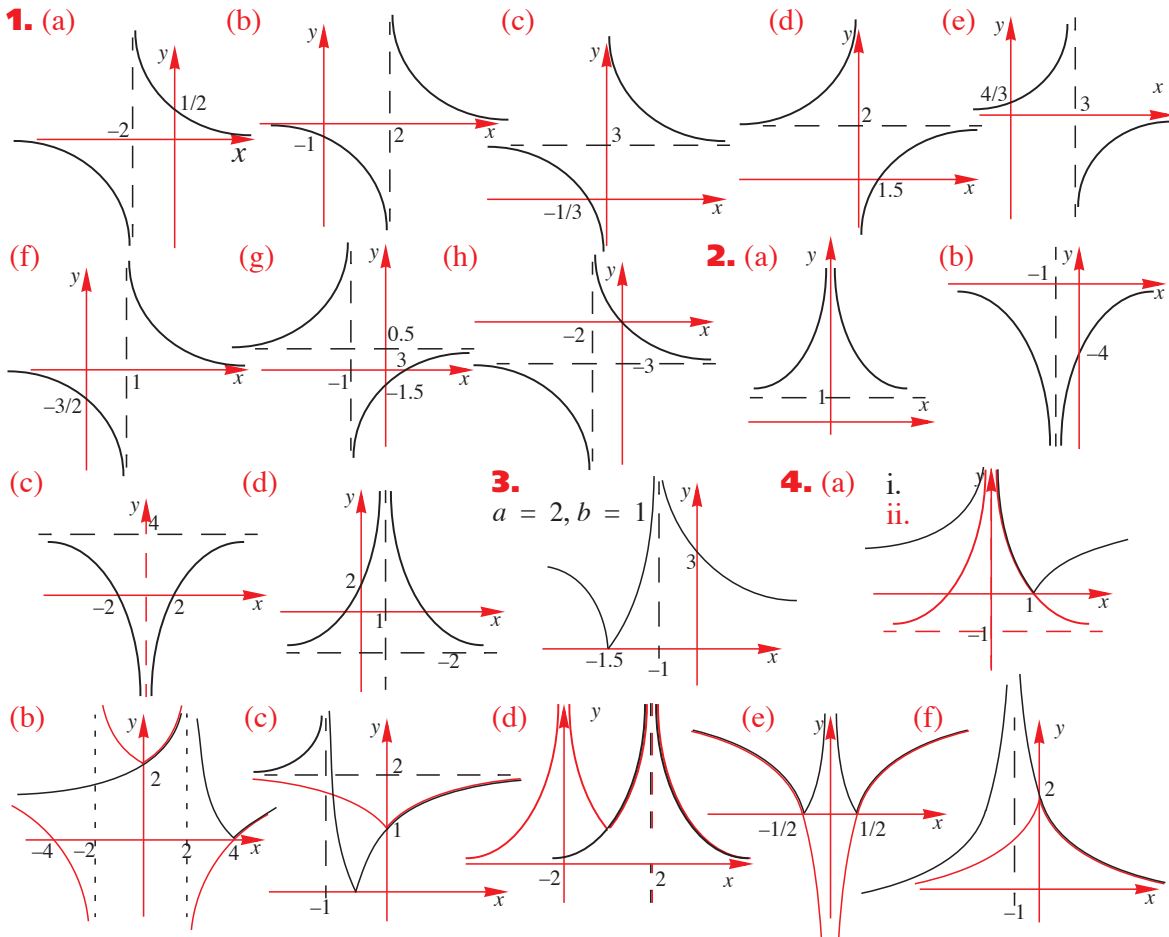
EXERCISE 5.3.4







EXERCISE 5.3.5



EXERCISE 5.4.1

- 1. (a) i.** $f + g: [0, \infty[\mapsto \mathbb{R}$ where $(f + g)(x) = x^2 + \sqrt{x}$ $[0, \infty[$
ii. $f + g:]0, \infty[\mapsto \mathbb{R}$ where $(f + g)(x) = \frac{1}{x} + \ln(x)$ $[1, \infty[$
iii. $f + g: [-3, -2] \cup [2, 3] \mapsto \mathbb{R}$ where $(f + g)(x) = \sqrt{9 - x^2} + \sqrt{x^2 - 4}$ $, [\sqrt{5}, \sqrt{10}]$
- (b) i.** $fg: [0, \infty[\mapsto \mathbb{R}$ where $(fg)(x) = x^2\sqrt{x} = x^{5/2}$
ii. $fg:]0, \infty[\mapsto \mathbb{R}$ where $(fg)(x) = \frac{\ln(x)}{x}$
iii. $fg: [-3, -2] \cup [2, 3] \mapsto \mathbb{R}$ where $(fg)(x) = \sqrt{(9 - x^2)(x^2 - 4)}$
- 2. (a) i.** $f - g:]-\infty, \infty[\mapsto \mathbb{R}$ where $(f - g)(x) = 2e^x - 1$ $]-1, \infty[$
ii. $f - g:]-1, \infty[\mapsto \mathbb{R}$ where $(f - g)(x) = (x + 1) - \sqrt{x + 1}$ $]-0.25, \infty[$
iii. $f - g:]-\infty, \infty[\mapsto \mathbb{R}$ where $(f - g)(x) = |x - 2| - |x + 2|$ $, [-4, 4]$
- (b) i.** $f/g: \mathbb{R} \setminus \{0\}, \mapsto \mathbb{R}$ where $(f/g)(x) = \frac{e^x}{1 - e^x}$

ii. $f/g:]-1, \infty[\mapsto \mathbb{R}$ where $(f/g)(x) = \sqrt{x+1}$

iii. $f/g: \mathbb{R} \setminus \{-2\} \mapsto \mathbb{R}$ where $(f/g)(x) = \left| \frac{x-2}{x+2} \right|$

3. i. (a) $fog(x) = x^3 + 1$, $gof(x) = (x+1)^3$ (b) $]-\infty, \infty[$, $]-\infty, \infty[$

ii. (a) $fog(x) = x+1$, $x \geq 0$, $gof(x) = \sqrt{x^2+1}$ (b) $[1, \infty[$, $[1, \infty[$

iii. (a) $fog(x) = x^2$, $gof(x) = (x+2)^2 - 2$ (b) $[0, \infty[$, $[-2, \infty[$

iv. (a) $fog(x) = x$, $x \neq 0$, $gof(x) = x$, $x \neq 0$ (b) $\mathbb{R} \setminus \{0\}$, $\mathbb{R} \setminus \{0\}$

v. (a) $fog(x) = x$, $x \geq 0$, $gof(x) = |x|$ (b) $[0, \infty[$, $[0, \infty[$

vi. (a) $fog(x) = \frac{1}{x^2} - 1$, $x \neq 0$, $gof(x)$ does not exist (b) $]-1, \infty[$

vii. (a) $fog(x) = x^2$, $x \neq 0$, $gof(x) = x^2$, $x \neq 0$ (b) $]0, \infty[$, $]0, \infty[$

viii. (a) $fog(x) = |x| - 4$, $gof(x) = |x-4|$ (b) $[-4, \infty[$, $[0, \infty[$

ix. (a) $fog(x) = |x+2|^3 - 2$, $gof(x) = |x^3|$ (b) $[-2, \infty[$, $[0, \infty[$

x. (a) $fog(x)$ does not exist, $gof(x) = (4-x)$, $x \leq 4$ (b) $[0, \infty[$

xi. (a) $fog(x) = \frac{x^2}{x^2+1}$, $gof(x) = \left(\frac{x}{x+1}\right)^2$, $x \neq -1$ (b) $[0, 1[$, $[0, \infty[$

xii. (a) $fog(x) = x^2 + |x| + 1$, $gof(x) = |x^2 + x + 1|$ (b) $[1, \infty[$, $[0.75, \infty[$

xiii. (a) $fog(x) = 2^{x^2}$, $gof(x) = 2^{2x}$ (b) $[1, \infty[$, $]0, \infty[$

xiv. (a) $fog(x)$ does not exist, $gof(x) = \frac{1}{x+1} - 1$, $x \neq -1$ (b) $\mathbb{R} \setminus \{-1\}$

xv. (a) $fog(x)$ does not exist, $gof(x) = \frac{4}{x-1} + 1$ (b) $]1, \infty[$

xvi. (a) $fog(x) = 4^{\sqrt{x}}$, $x \geq 0$, $gof(x) = 4^{0.5x}$ (b) $[1, \infty[$, $]0, \infty[$

4. (a) $fog(x) = 2x+3$, $x \in \mathbb{R}$ (b) $gof(x) = 2x+2$, $x \in \mathbb{R}$ (c) $fof(x) = 4x+3$, $x \in \mathbb{R}$

5. $g(x) = x^2 + 1$, $x \in \mathbb{R}$ 6. (a) $fog(x) = \frac{1}{x} + x + 1$, $x \in \mathbb{R} \setminus \{0\}$, $]-\infty, -1] \cup [3, \infty[$

(b) $gof(x)$ does not exist. (c) $gog(x) = x + \frac{1}{x} + \frac{x}{x^2+1}$, $x \neq 0$, $]-\infty, -2.5] \cup [2.5, \infty[$

7. (a) 9 (b) 3 9. (a) $x = \pm 1$ (b) $x = 1, -3$ 10. (a) $\frac{1}{x}$ (b) $\frac{-x}{2x+1}$

11. $hof(x) = \begin{cases} (x-1)^2 + 4, & x \geq 2 \\ 5-x, & x < 2 \end{cases}$ 12. (a) $r_f \subseteq d_g$ and $r_{gof} \subseteq d_h$ (b) $g(x) = 4(x+1)^2$, $x \in \mathbb{R}$

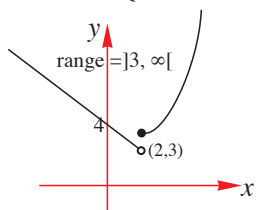
13. (a) $fog(x) = x$, $x \in]0, \infty[$ range = $]0, \infty[$ (b) $gof(x) = \frac{1}{2}(\ln(e^{2x-1}) + 1)$, $x \in \mathbb{R}$ (= x) range = $]-\infty, \infty[$

(c) $fof(x) = e^{2(e^{2x-1})-1}$, $x \in \mathbb{R}$ range = $]e^{-1}, \infty[$

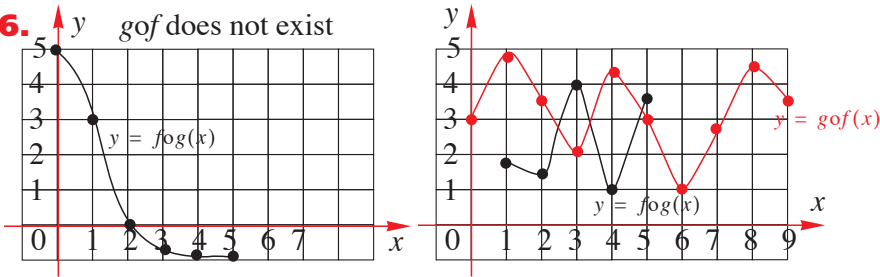
14. (a) hok does not exist.

(b) $koh(x) = 4\log(4x-1) - 1$, $x > \frac{1}{4}$, \mathbb{R}

15. (a) $S = \mathbb{R} \setminus]-3, 3[$; $T = \mathbb{R}$ (b) $T = \{x : |x| \geq 6, x = 0\}$; $S =]-\infty, -3] \cup [3, \infty[$



16. y gof does not exist

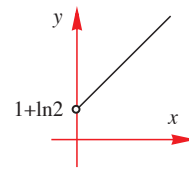


17. (a) $\text{Dom } f =]0, \infty[$, $\text{ran } f =]e, \infty[$, $\text{Dom } g =]0, \infty[$, $\text{ran } g = \mathbb{R}$

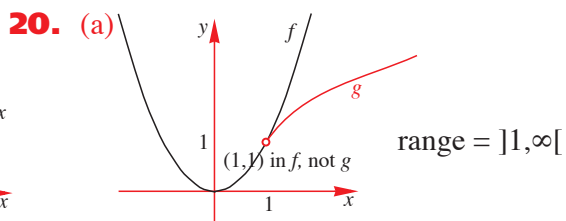
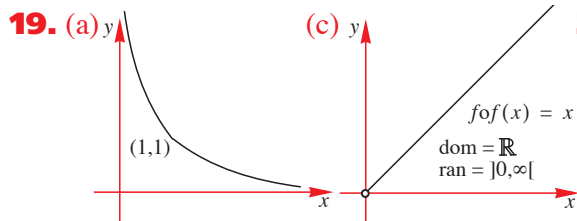
(b) gof does not exist: $r_g = \mathbb{R} \not\subseteq d_f =]0, \infty[$

gof exists as $r_f =]e, \infty[\subseteq d_g =]0, \infty[$

(c) $gof:]0, \infty[\mapsto \mathbb{R}$, where $gof(x) = (x+1) + \ln 2$



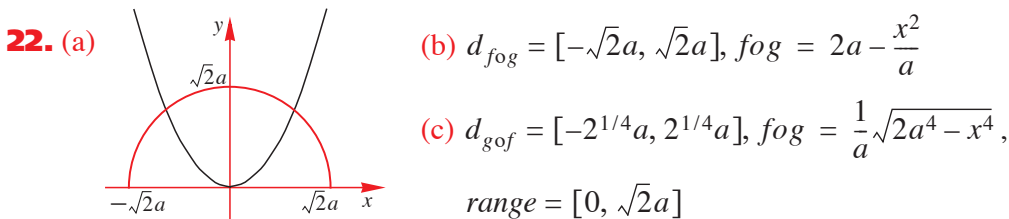
18. $(f \circ g)(x) = |x|$, $x \in \mathbb{R}$; range = $[0, \infty[$



(b) $gof:]1, \infty[\mapsto \mathbb{R}$, where $gof(x) = x$

(d) $f \circ g^*:]1, \infty[\mapsto \mathbb{R}$, where $gof(x) = x$

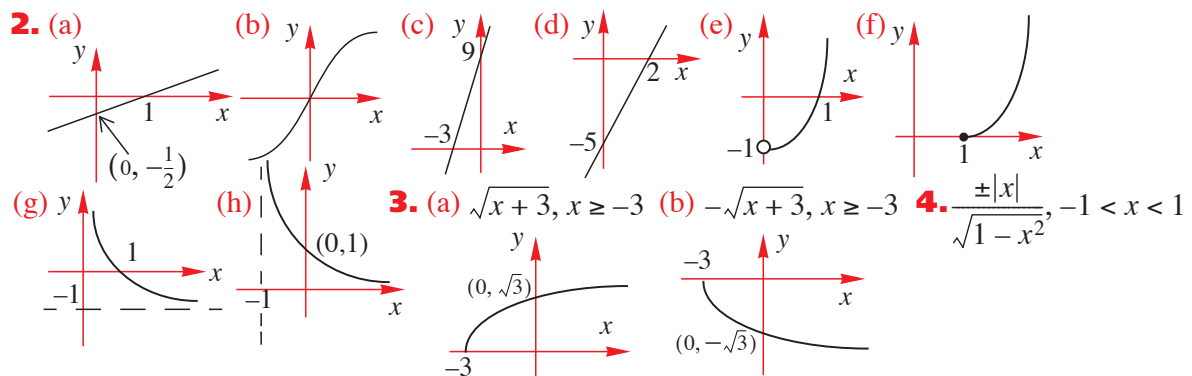
21. $d_f = \mathbb{R} \setminus \left\{ \frac{a}{c} \right\}$, $r_f = \mathbb{R} \setminus \left\{ \frac{a}{c} \right\}$, $r_f \subseteq d_f$, $f \circ f(x) = x$

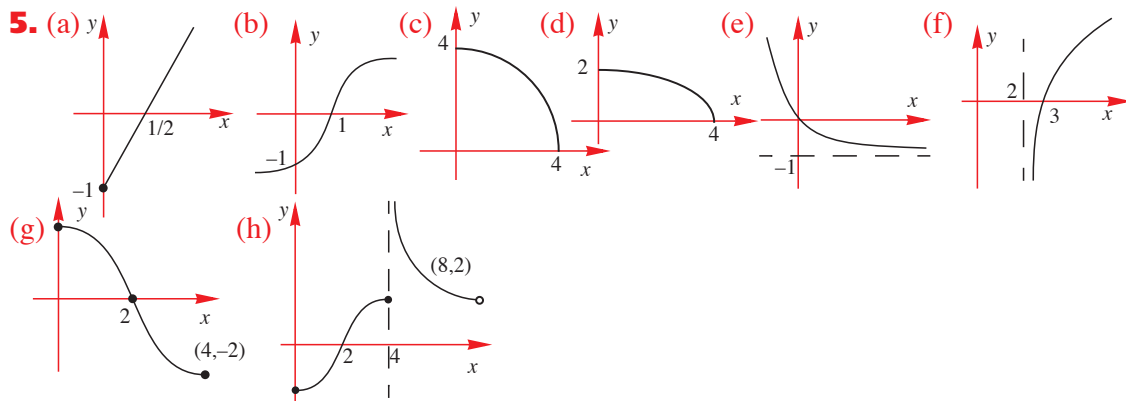


EXERCISE 5.4.2

1. (a) $\frac{1}{2}(x-1)$, $x \in \mathbb{R}$ (b) $\sqrt[3]{x}$, $x \in \mathbb{R}$ (c) $3(x+3)$, $x \in \mathbb{R}$ (d) $\frac{5}{2}(x-2)$, $x \in \mathbb{R}$ (e) $x^2 - 1$, $x > 0$

(f) $(x-1)^2$, $x \geq 1$ (g) $\frac{1}{x} - 1$, $x > 0$ (h) $\frac{1}{(x+1)^2}$, $x > -1$

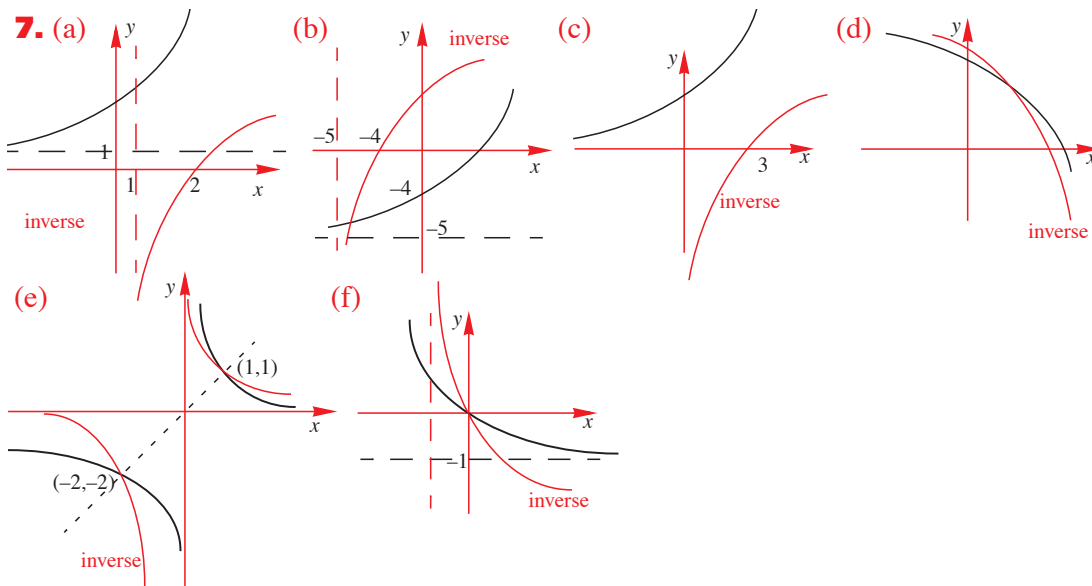




6. (a) $f^{-1}(x) = \log_3(x-1), x > 1$ **(b)** $f^{-1}(x) = \log_2(x+5), x > -5$

(c) $f^{-1}(x) = \frac{1}{2}(\log_3 x - 1), x > 0$ **(d)** $g^{-1}(x) = 1 + \log_{10}(3-x), x < 3$

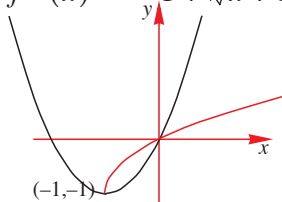
(e) $h^{-1}(x) = \log_3\left(1 + \frac{2}{x}\right), x \in \mathbb{R} \setminus [-2, 0]$ **(f)** $g^{-1}(x) = \log_2\left(\frac{1}{x+1}\right), x > -1$



8. (a) $f^{-1}(x) = 2^x - 1, x \in \mathbb{R}$ **(b)** $f^{-1}(x) = \frac{1}{2} \cdot 10^x, x \in \mathbb{R}$ **(c)** $h^{-1}(x) = 2^{1-x}, x \in \mathbb{R}$

(d) $g^{-1}(x) = 3^{x+1} + 1, x \in \mathbb{R}$ **(e)** $h^{-1}(x) = 5^{x/2} + 5, x \in \mathbb{R}$ **(f)** $f^{-1}(x) = 1 - 10^{3(2-x)}, x \in \mathbb{R}$

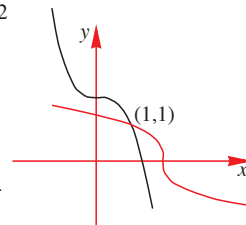
9. $f^{-1}(x) = \frac{y}{1} + \sqrt{x+1}, x > -1$ **10. (a)** $f^{-1}(x) = a - x$ **(b)** $f^{-1}(x) = \frac{2}{x-a} + a$



dom = $[-1, \infty[$, ran = $[-1, \infty[$

(c) $f^{-1}(x) = \sqrt{a^2 - x^2}$

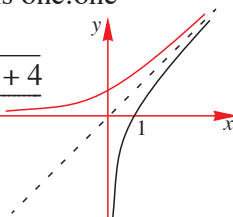
11. $f^{-1}(x) = \sqrt[3]{2-x}$



14. (a) Inverse exists as f is one:one

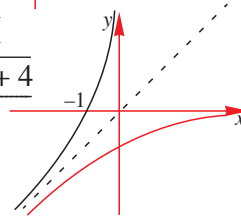
(b) Case 1: $S =]0, \infty[$

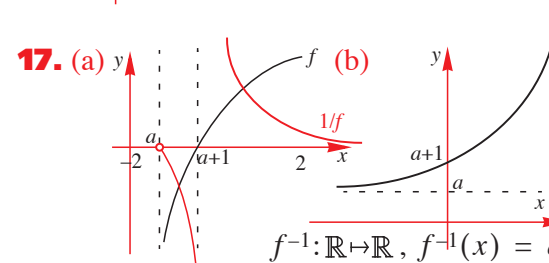
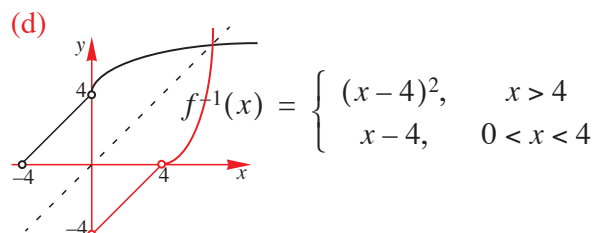
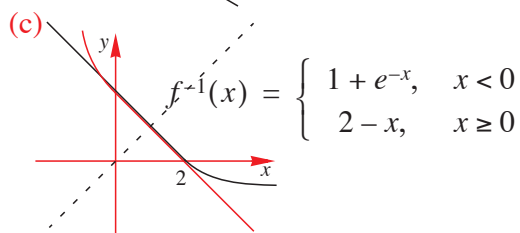
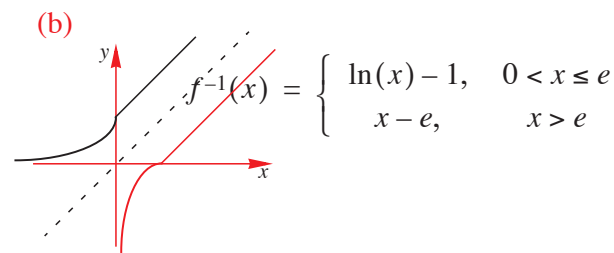
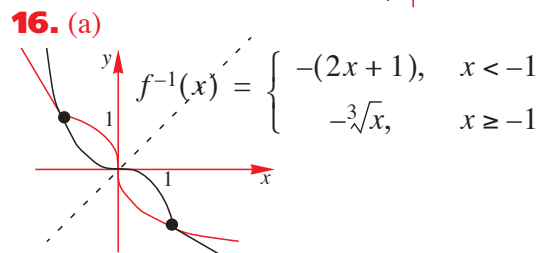
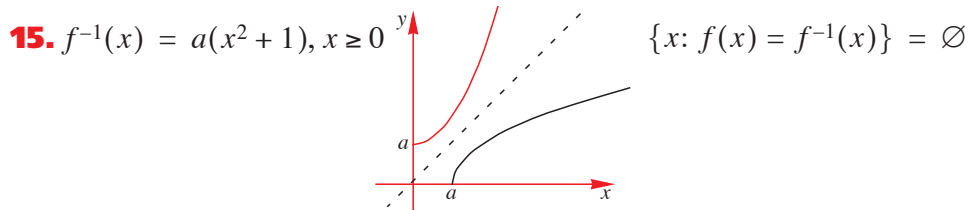
$g^{-1}(x) = \frac{x + \sqrt{x^2 + 4}}{2}$



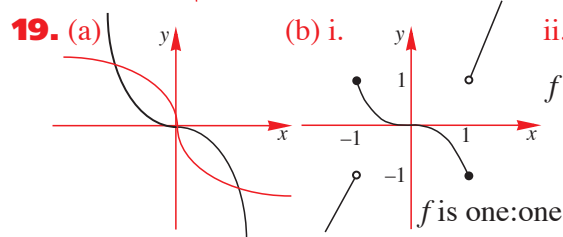
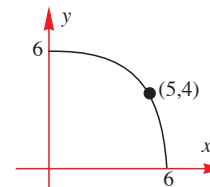
Case 2: $S =]-\infty, 0[$

$g^{-1}(x) = \frac{x - \sqrt{x^2 + 4}}{2}$





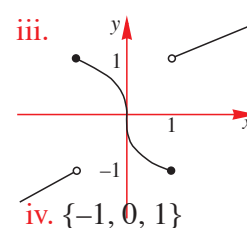
18. $g \circ f$ exists as $r_f \subseteq d_g$. It is one:one so the inverse exists:



$f^{-1}: \mathbb{R} \rightarrow \mathbb{R}, f^{-1}(x) = a + e^{ax}$

(b) i. $f(x) = \begin{cases} \frac{1}{2}(x-1) & x < -1 \\ -\sqrt[3]{x} & -1 \leq x \leq 1 \\ \frac{1}{2}(x+1) & x > 1 \end{cases}$

ii. f is one:one



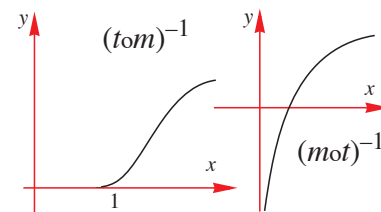
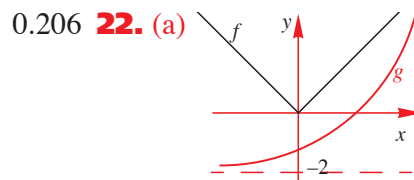
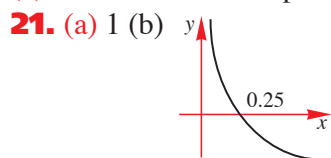
20. (a) i. $tom(x) = e^{\sqrt{x}}, x \geq 0$ **ii.** $mot(x) = \sqrt{e^x}, x \in \mathbb{R}$ **(b) i.** $(tom)^{-1}(x) = (\ln(x))^2, x > 1$

ii. $(mot)^{-1}(x) = \ln x^2, x > 0$ **(c) i. & ii.** neither exist

(d) Adjusting domains so that the functions in (c) exist, we

have: $t^{-1}om^{-1}(x) = (mot)^{-1}(x)$ & $m^{-1}ot^{-1}(x) = (tom)^{-1}(x)$

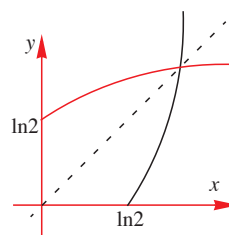
(e) Yes as rules of composition OK.



(b) $f \circ g$ exists but is not one:one

(c) i. $B = [\ln 2, \infty[$

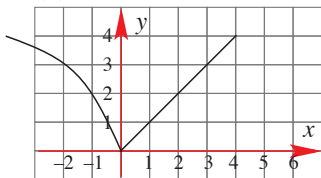
ii. $(f \circ g)^{-1}: [0, \infty[\rightarrow \mathbb{R}$ where, $(f \circ g)^{-1}(x) = \ln(x+2)$ **iii.**



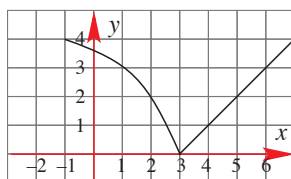
EXERCISE 6.1

1. (a) $y = (x-4)^2$ (b) $y = (x+2)^2$ (c) $y = x^2 + 5$ (d) $(x-2)^2 + y = 2$ (e) $x^2 + y = 4$
 (f) $x^2 + y = 0$ (g) $y = \frac{8}{x-4}, x \neq 4$ (h) $y = \frac{8}{x} - 1, x \neq 0$ (i) $(x+1)^2 + y^2 = 4$
 (j) $y^2 = \frac{9}{x-3}, x \neq 3$ (k) $(y+3)^2 = \frac{9}{x}, x \neq 0$ (l) $x + y^2 = 8$

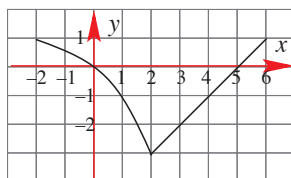
2. (a) i



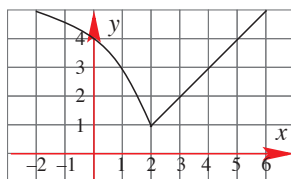
(a) ii



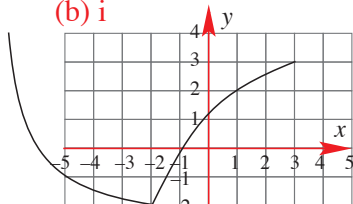
(a) iii



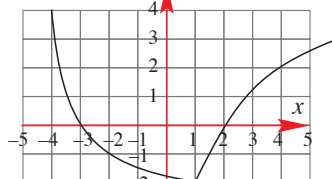
(a) iv



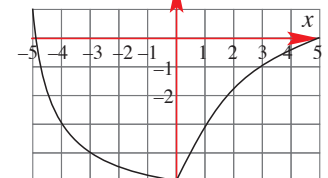
(b) i



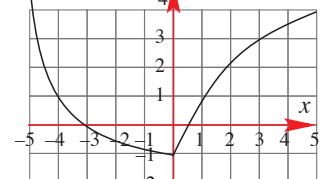
(b) ii



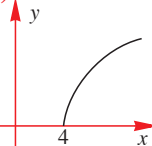
(b) iii



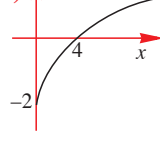
(b) iv



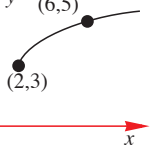
3. (a)



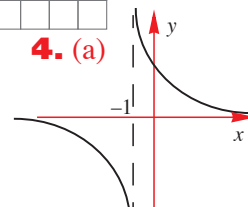
(b)



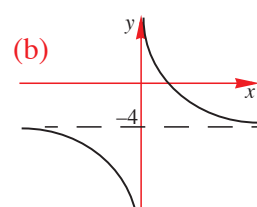
(c)



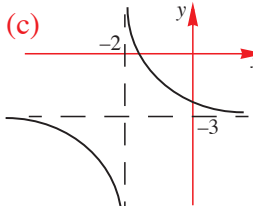
4. (a)



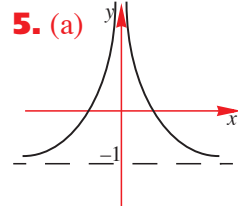
(b)



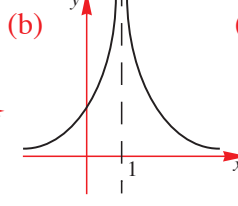
(c)



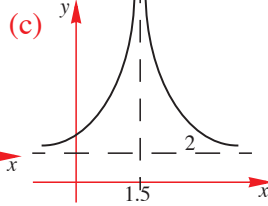
5. (a)



(b)

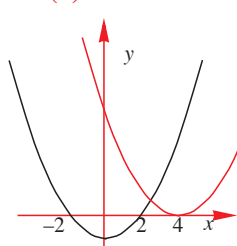


(c)

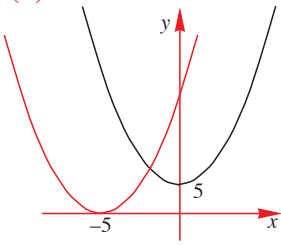


6. First function in black, second function in red

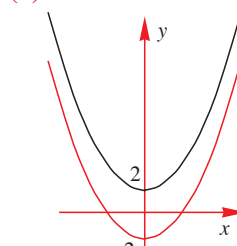
(a)



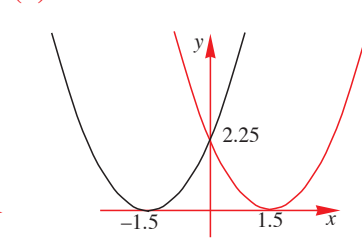
(b)

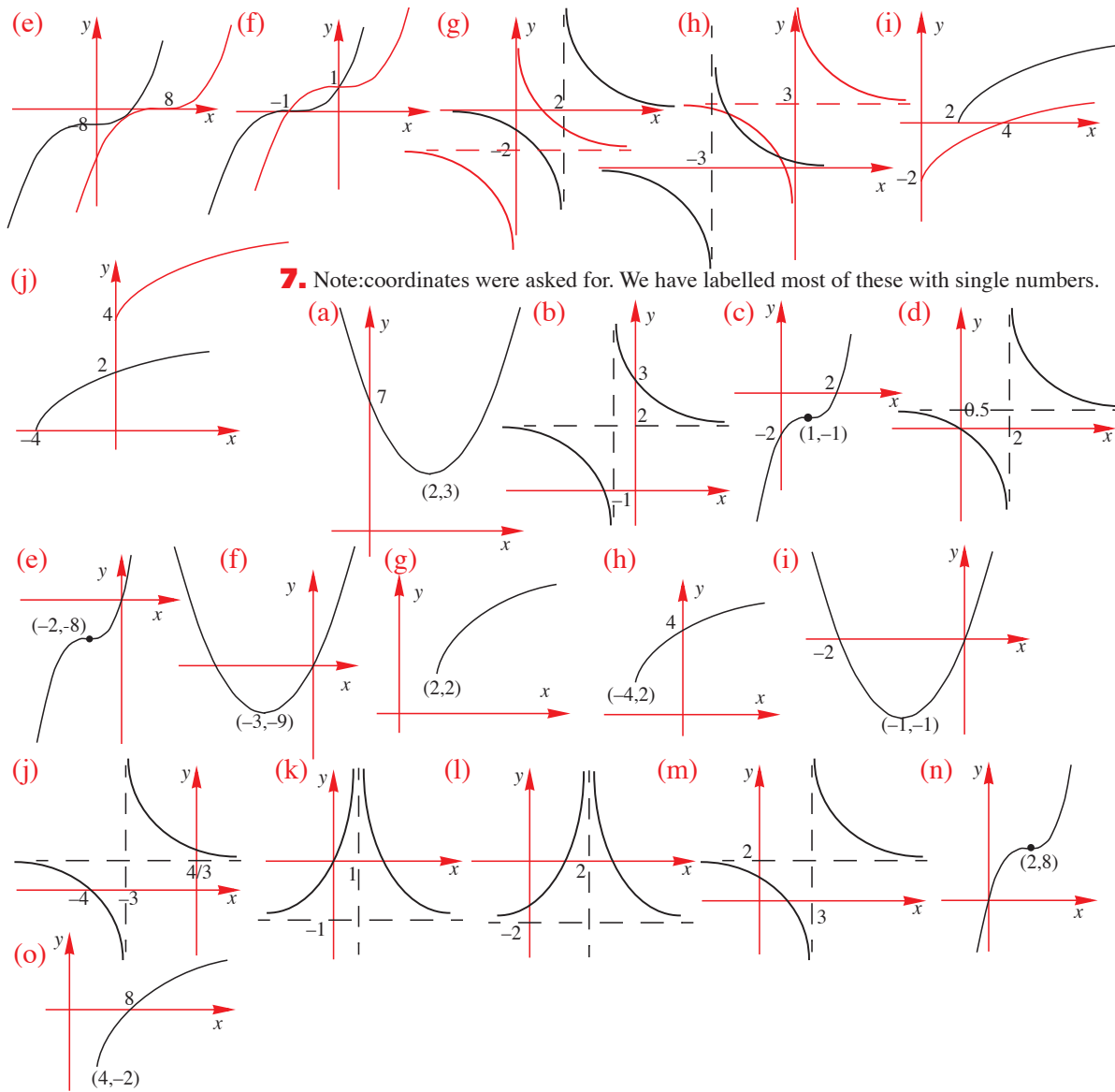


(c)



(d)



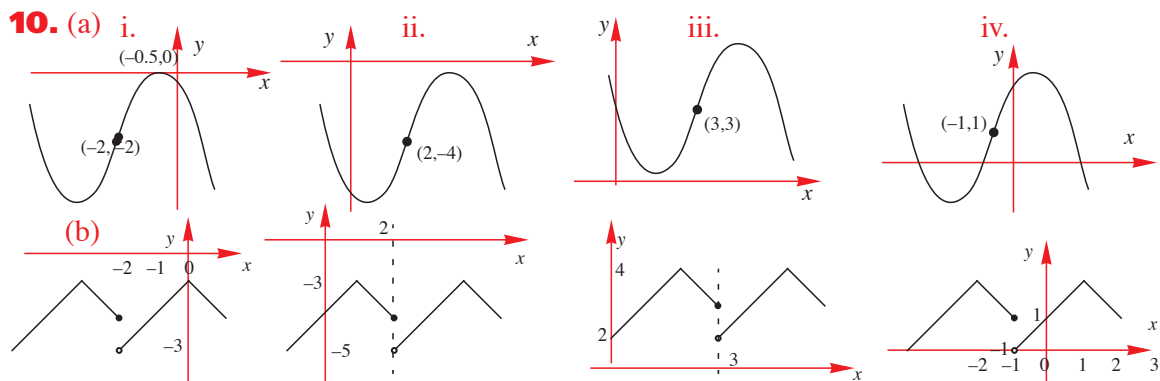


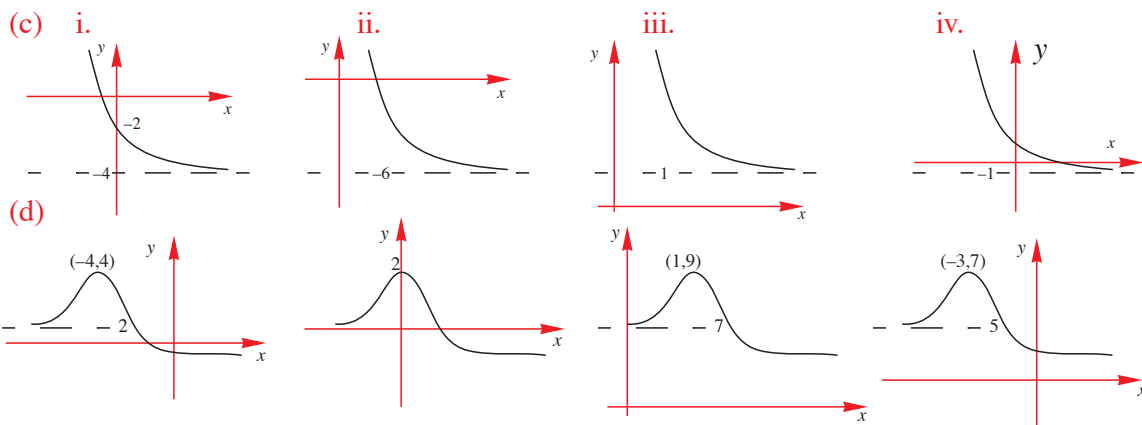
7. Note: coordinates were asked for. We have labelled most of these with single numbers.

8. (a) $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ (e) $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ (f) $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ (g) $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ (h) $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

(i) $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ (j) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (k) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ (l) $\begin{pmatrix} -k \\ h \end{pmatrix}$ (m) $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ (n) $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (o) $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

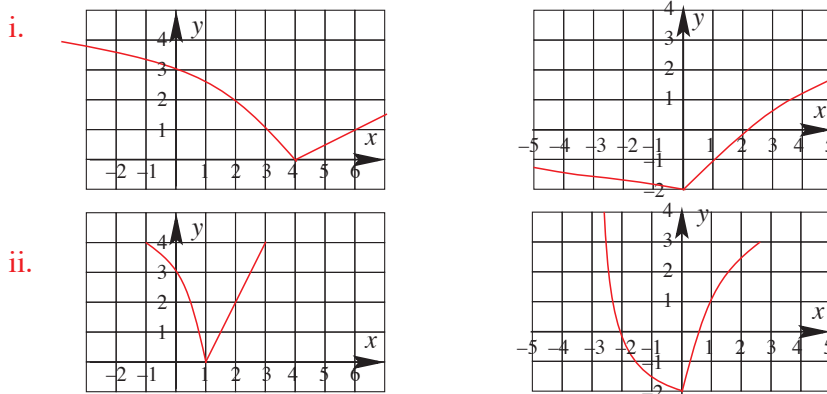
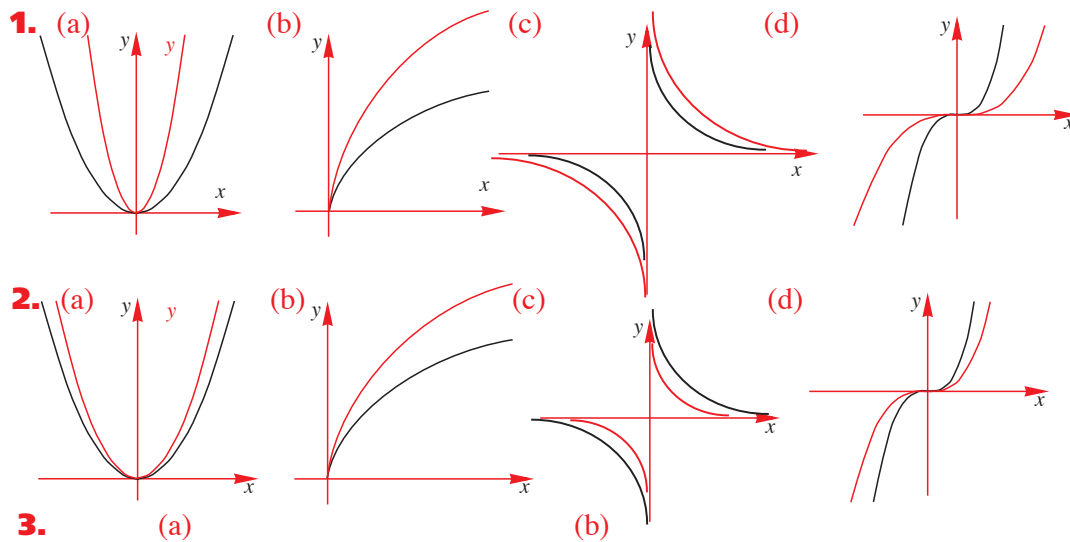
9. (a) $g(x) = f(x-1) + 1$ (b) $g(x) = f(x+2) - 4$ (c) $g(x) = f(x-2)$
 (d) $g(x) = f(x-1) + 1$ (e) $g(x) = f(x-1) + 3$

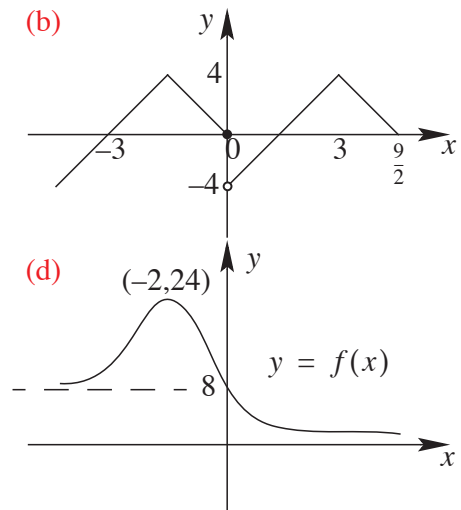
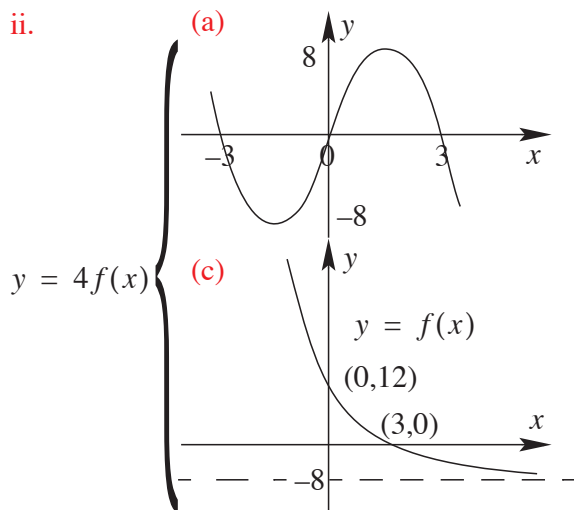
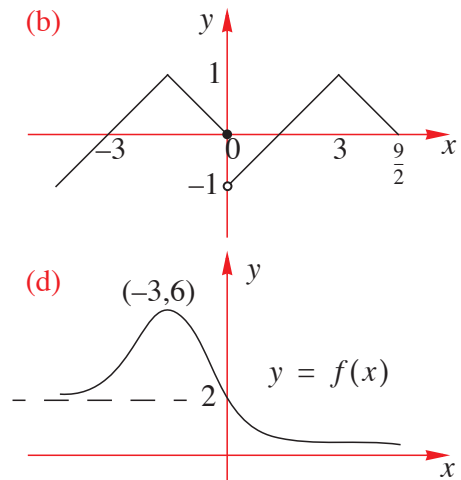
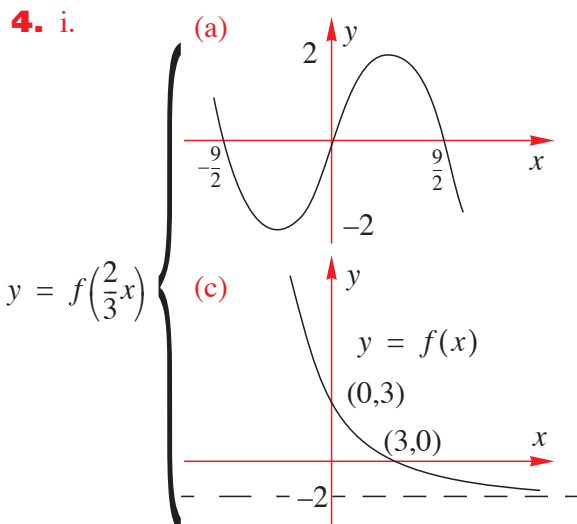
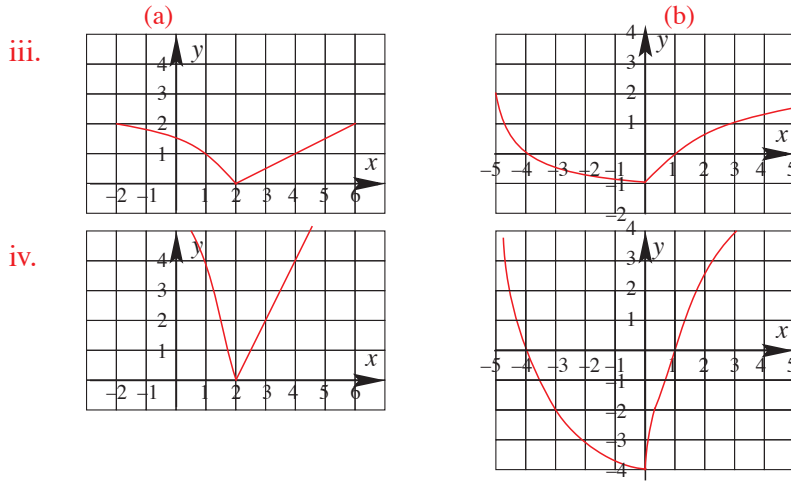




11. $y = \begin{cases} f(x+2) + 2, & -3 \leq x \leq -1 \\ f(x+4) + 2, & -5 \leq x \leq -3 \end{cases}$

EXERCISE 6.2



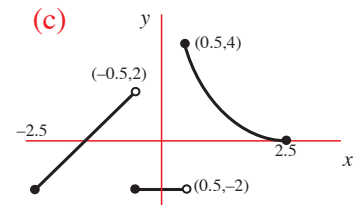
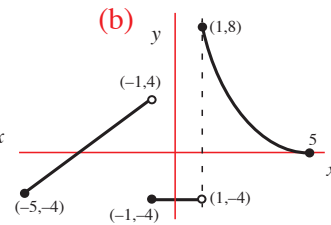
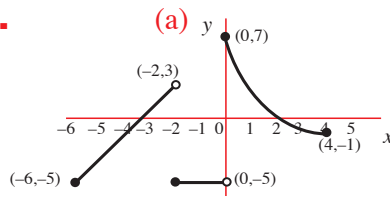


5. (a) $f(x) = |x|$ $y = f(2x) + 1$ (b) $f(x) = x^2$ $y = \frac{1}{2}f(x-2) - 3$

(c) $f(x) = \frac{1}{x}$ $y = \frac{1}{2}f\left(x - \frac{1}{2}\right)$ (d) $f(x) = x^3$ $y = 27f\left(x - \frac{2}{3}\right)$

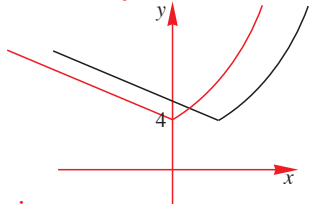
(e) $f(x) = x^4$ $y = 128f\left(x - \frac{1}{2}\right) - 2$ (f) $f(x) = \sqrt{x}$ $y = \sqrt{2}f(x) + 2$

6.



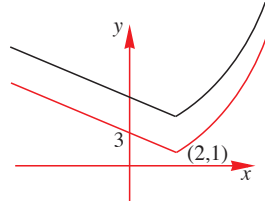
7. i.

$$f(x) = \begin{cases} (x+2)^2 & \text{if } x \geq 0 \\ 4-x & \text{if } x < 0 \end{cases}$$



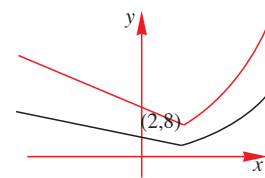
ii.

$$h(x) = \begin{cases} x^2 - 3 & \text{if } x \geq 2 \\ 3 - x & \text{if } x < 2 \end{cases}$$



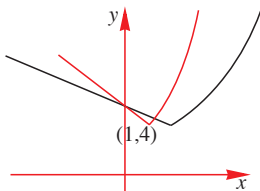
iii.

$$h(x) = \begin{cases} 2x^2 & \text{if } x \geq 2 \\ 12 - 2x & \text{if } x < 2 \end{cases}$$



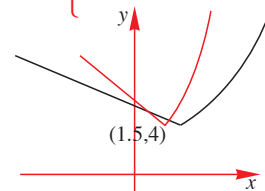
iv.

$$k(x) = \begin{cases} 4x^2 & \text{if } x \geq 1 \\ 6 - 2x & \text{if } x < 1 \end{cases}$$



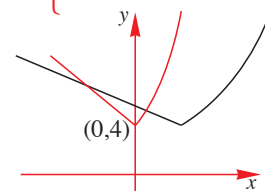
v.

$$k(x) = \begin{cases} (2x-1)^2 & \text{if } x \geq \frac{3}{2} \\ 7 - 2x & \text{if } x < \frac{3}{2} \end{cases}$$

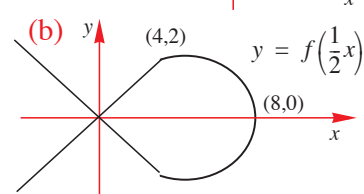
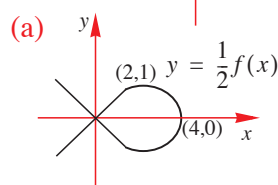
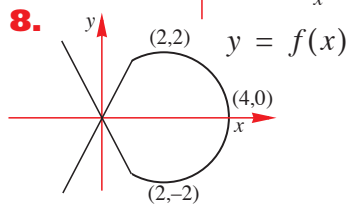


vi.

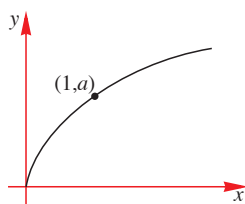
$$f(x) = \begin{cases} \frac{1}{2}(4x+2)^2 & \text{if } x \geq 0 \\ 2 - 2x & \text{if } x < 0 \end{cases}$$



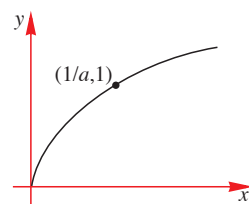
8.



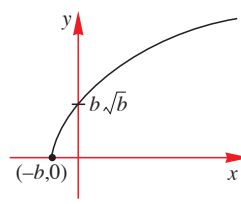
9. (a)



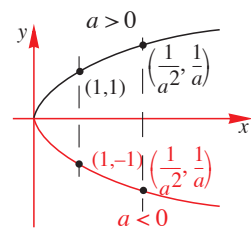
(b)



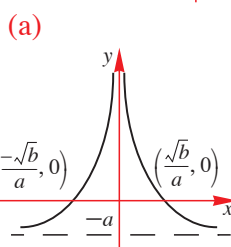
(c)



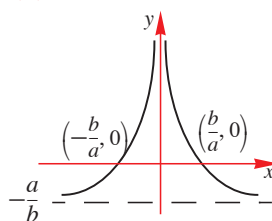
(d)



10.

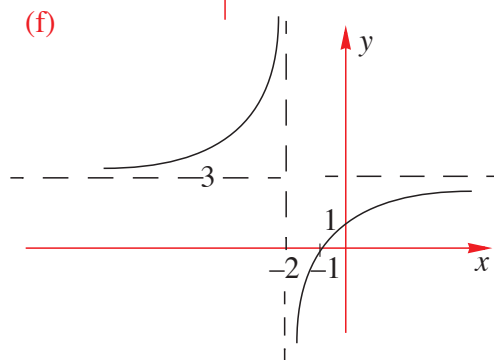
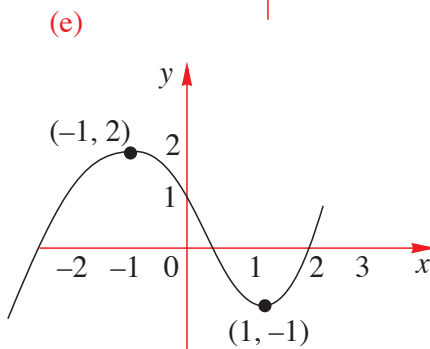
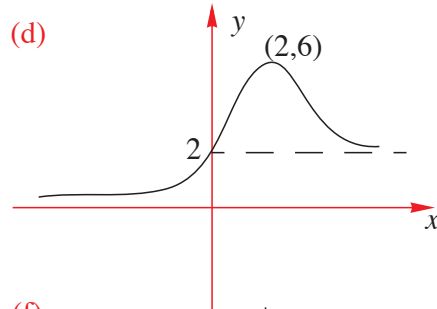
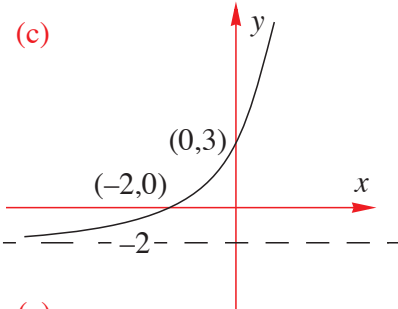
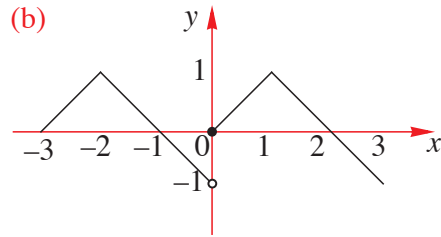
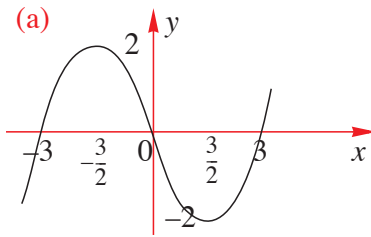


(b)

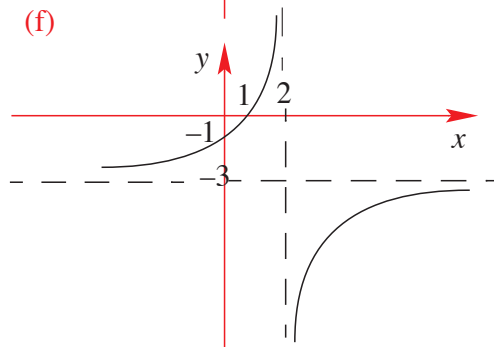
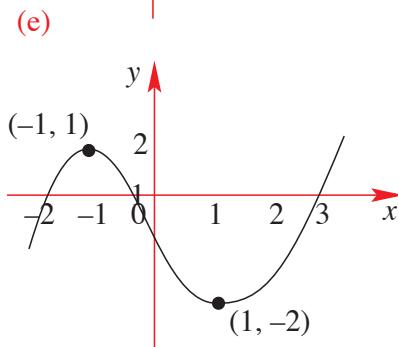
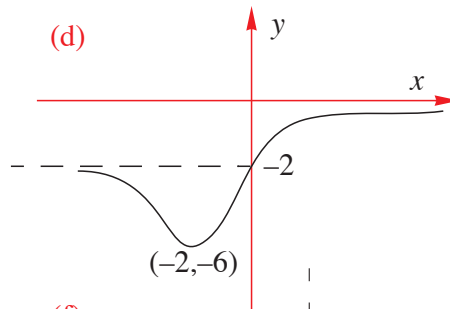
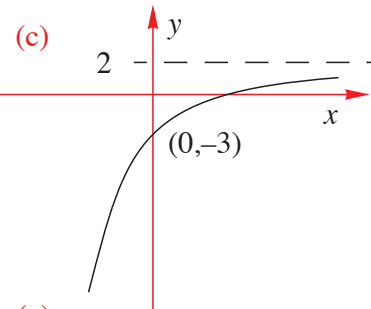
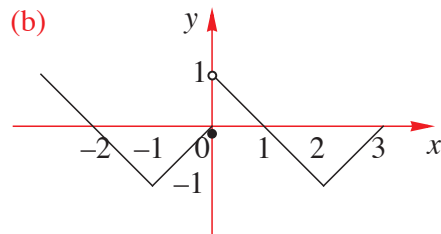
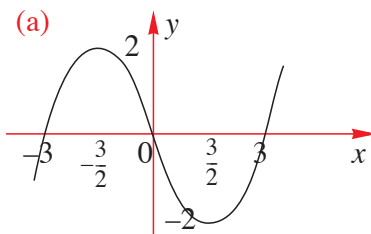


EXERCISE 6.3

1. i.

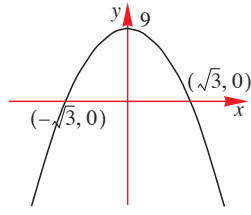


ii.



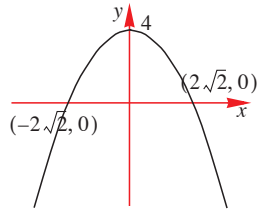
2. (a) $y = -f(x)$ (b) $y = f(-x)$ (c) $y = f(x+1)$ (d) $y = f(2x)$ (e) $y = 2f(x)$

3. (a)



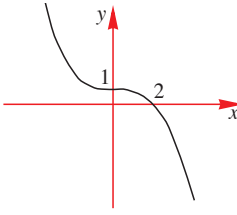
(e)

(b)



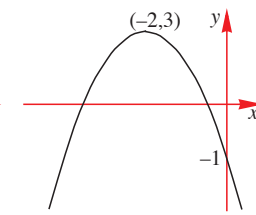
(f)

(c)

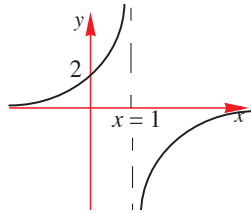


(g)

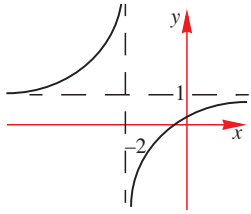
(d)



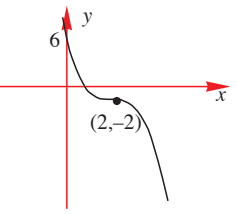
(h)



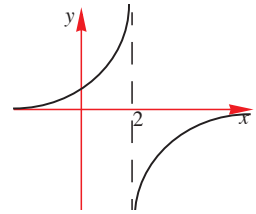
(i)



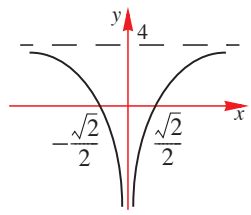
(j)



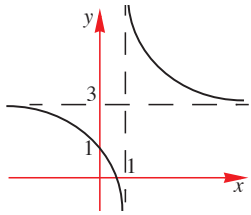
(k)



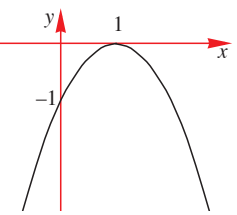
(l)



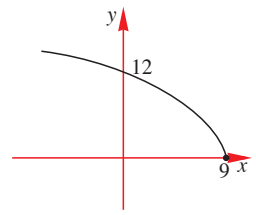
(m)



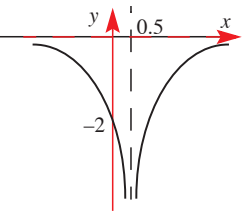
(n)



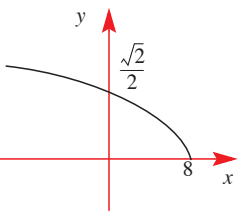
(o)



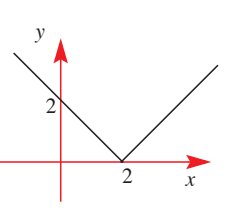
(p)



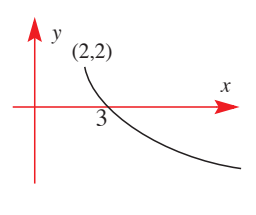
(q)



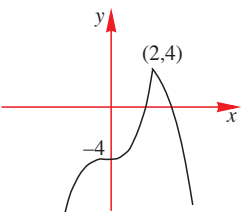
(r)



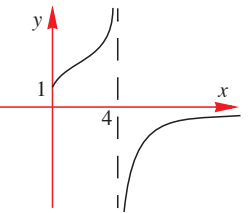
4. (a)



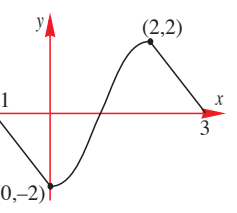
(b)



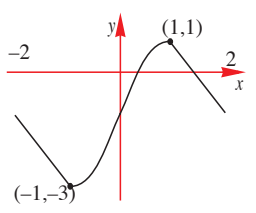
(c)



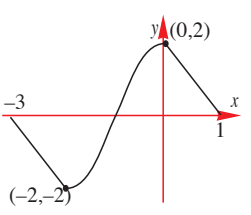
(d)



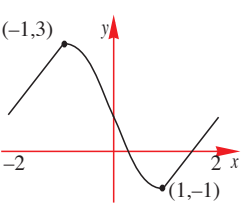
(e)



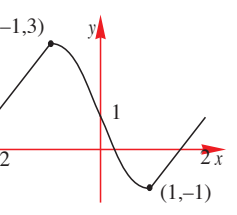
(f)



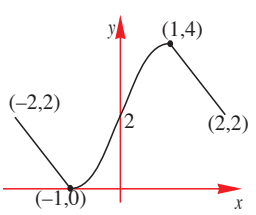
(g)



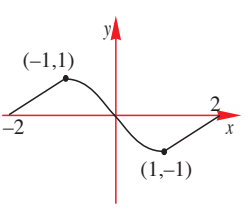
(h)



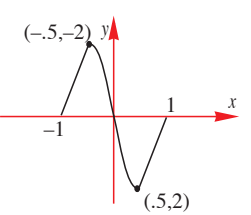
(i)



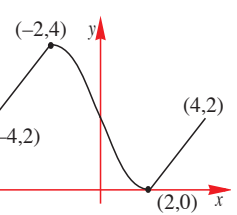
(j)



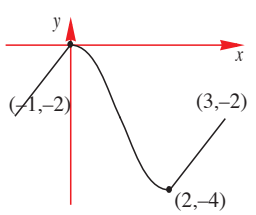
(k)



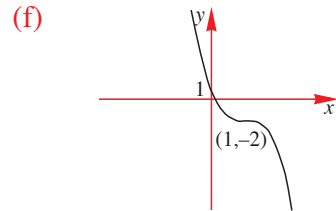
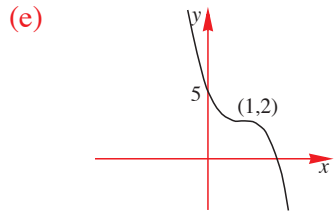
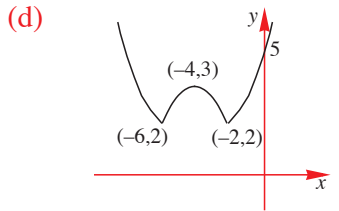
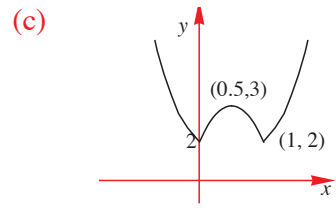
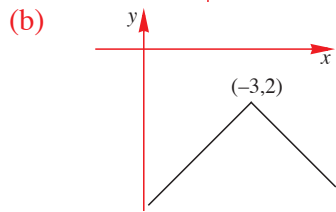
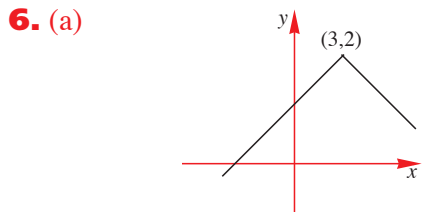
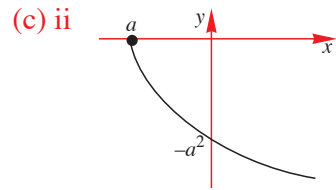
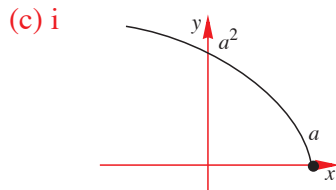
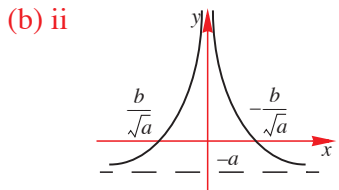
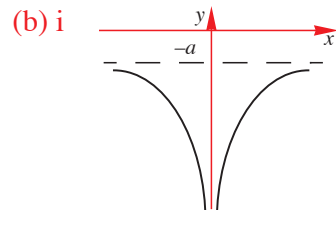
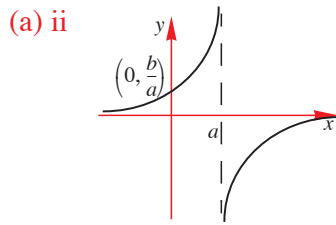
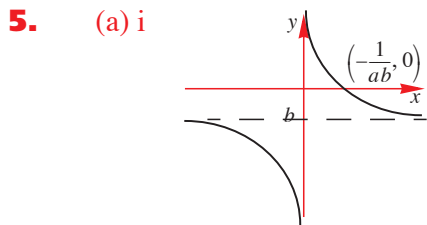
(l)



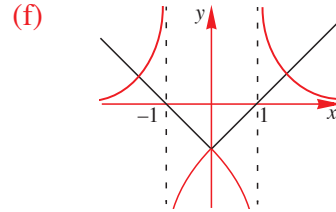
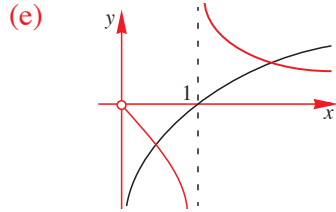
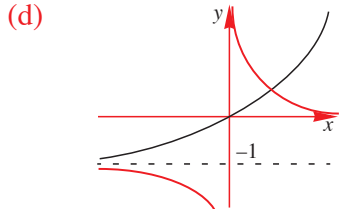
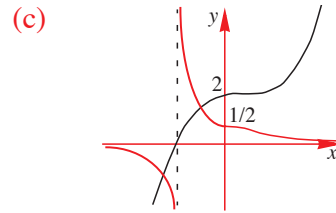
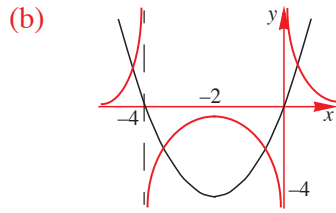
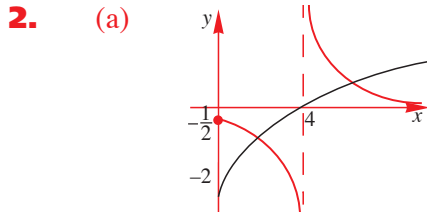
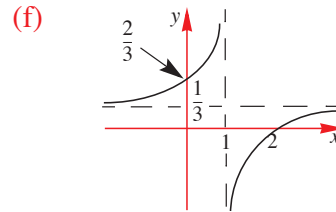
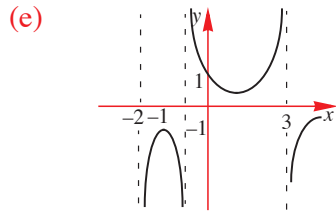
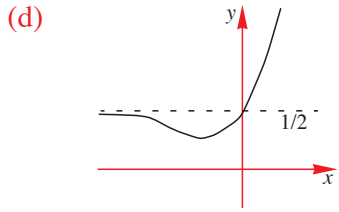
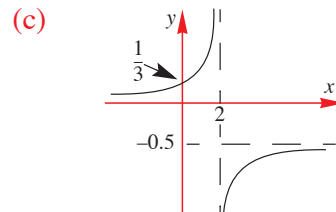
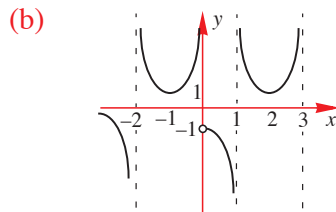
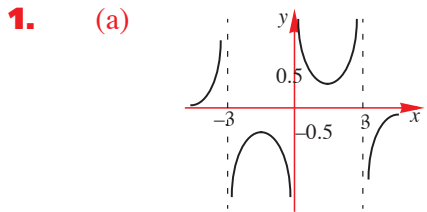
(m)

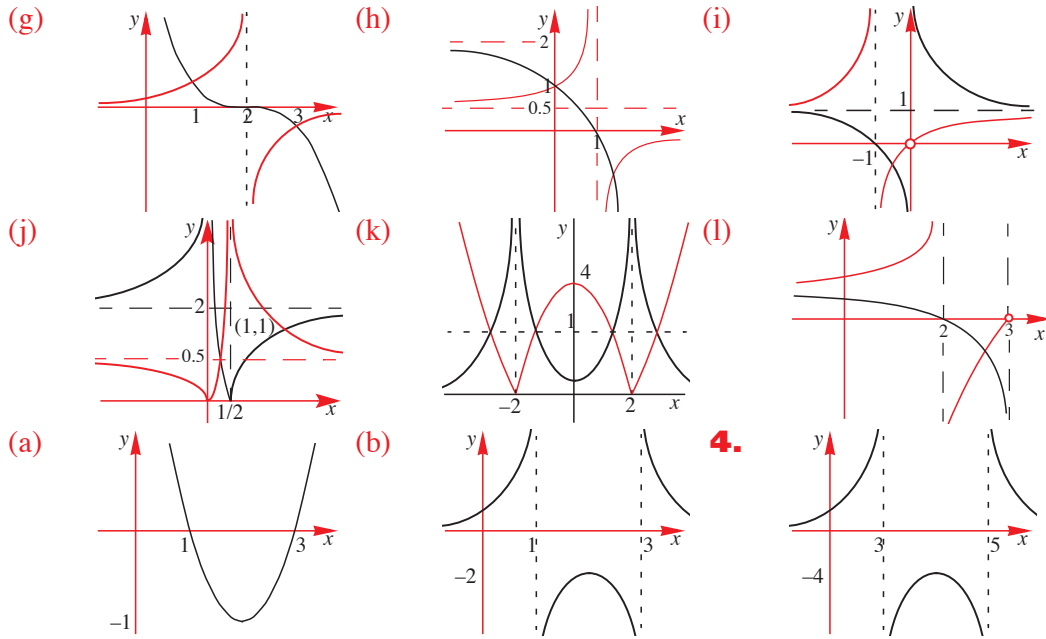


(n)

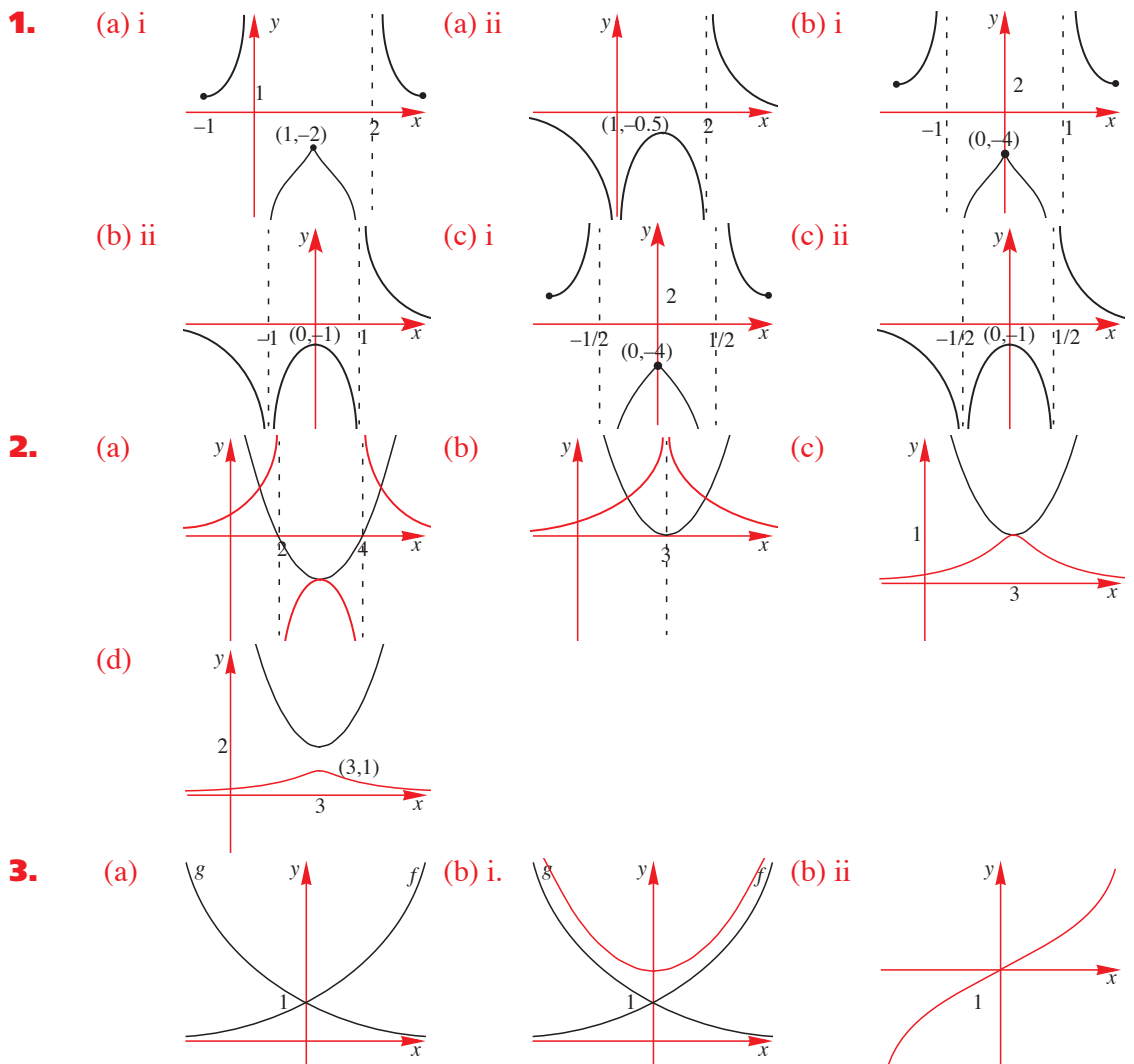


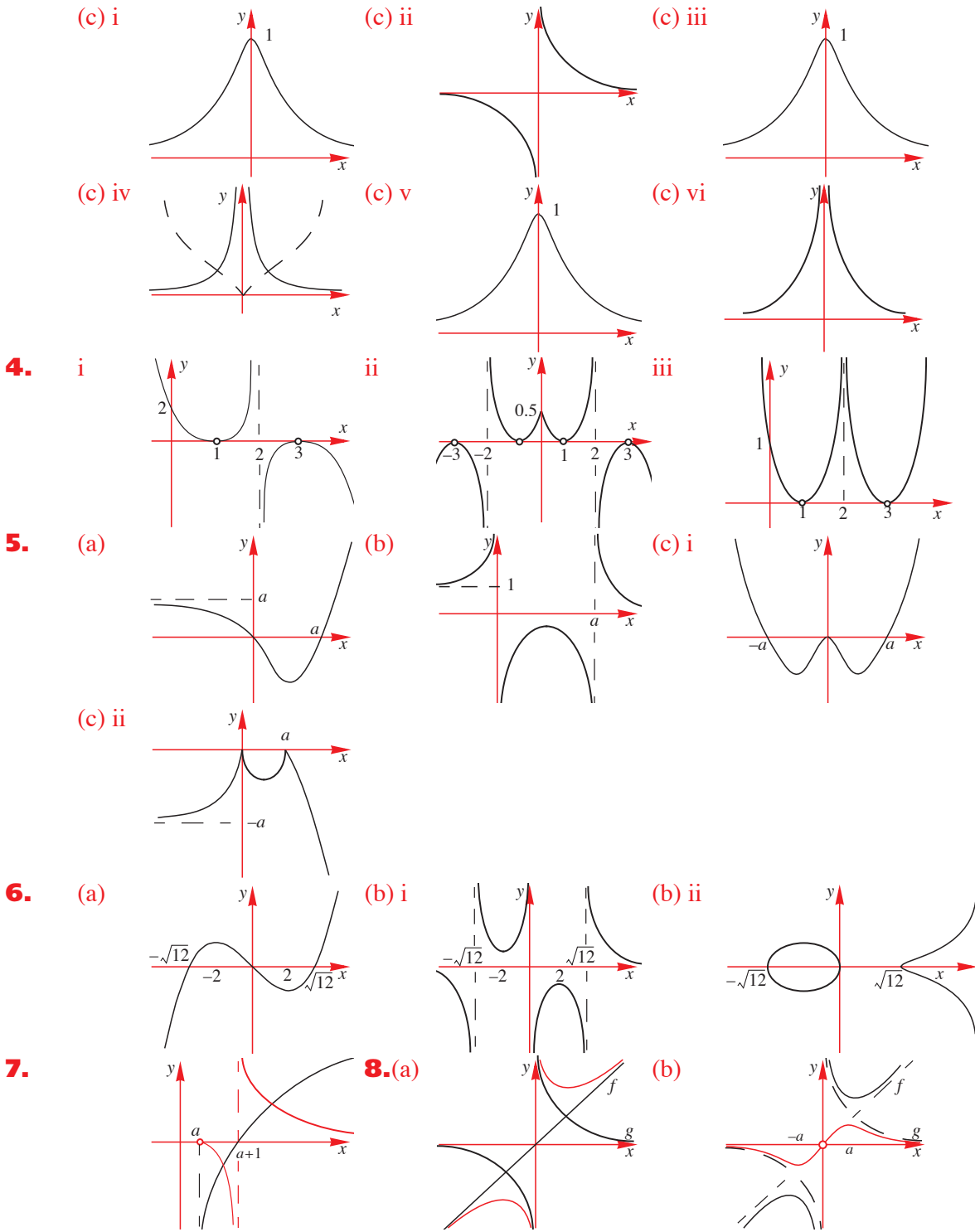
EXERCISE 6.4





EXERCISE 6.5





EXERCISE 7.1.1

1. (a) $\frac{27y^{15}}{8x^3}$ (b) $\frac{91}{216a^6}$ (c) $2^n + 2$ (d) $\frac{8x^{11}}{27y^2}$ (e) $\frac{3x^2y^2}{8}$ (f) $3^{n+1} + 3$ (g) $4^{n+1} - 4$
- (h) $2(4^{n+1} - 4)$ (i) $\frac{1-b^6}{16b^4}$ 2. (a) 64 (b) $\left(\frac{2}{3}\right)^x$ (c) 2^{2y+1} (d) $\frac{1}{b^{2x}}$ (e) $\left(\frac{y}{2}\right)^6$ (f) $\left(\frac{9}{2}\right)^{n+2}$ 3. (a) $\frac{z^2}{xy}$
- (b) 3^{7n-2} (c) 5^{n+1} (d) 9 (e) 2^{6n+1} (f) 2^{1-3n} (g) x^{2+4n-n^2} (h) x^{3n^2+n+1} (i) 27 4. $\frac{y^{2m-2}}{x^m}$
5. (a) -81 (b) $-\frac{9x^8}{8y^4}$ (c) $y - x$ (d) $\frac{2x+1}{x+1}$ (e) -1 (f) -b 6. (a) $\frac{1}{x^2y^2}$ (b) $\frac{1}{x^4}$ (c) $-\frac{1}{x(x+h)}$

(d) $\frac{1}{x-1}$ (e) $\frac{1}{(x+1)(x-1)^5}$ (f) $\frac{1}{x^2}$ **7.** (a) $118 \times 5^{n-2}$ (b) 1 (c) $\frac{b^7}{a^4}$ (d) a^{mn} (e) $\frac{p+q}{pq}$ (f) $\frac{2\sqrt{a}}{a-1}$
 (g) $\frac{7}{8}$ (h) $a^{7/8}$ **8.** (a) $x^{11/12}$ (b) $2a^{3n-2}b^{2n-2}$ (c) 2^n (d) $-\frac{7^{m-n}}{8}$ (e) $\frac{6 \times 5^n}{5^n + 5}$ (f) $x+1$

EXERCISE 7.1.2

1. (a) 2 (b) -2 (c) $\frac{2}{3}$ (d) 5 (e) 6 (f) -2.5 (g) 2 (h) 1.25 (i) $\frac{1}{3}$ **2.** (a) -6 (b) $-\frac{2}{3}$ (c) -3 (d) 1.5 (e) 0.25
 (f) 0.25 (g) $-\frac{1}{8}$ (h) $-\frac{11}{4}$ (i) -1.25

EXERCISE 7.1.3

1. (a) 3.5 (b) 3.5 (c) -3 (d) 1.5 (e) 3.5 (f) 1.5 (g) 1.8 (h) $-\frac{4}{7}$ (i) 0 **2.** (a) -0.75 (b) -1,4 (c) 0,1
 (d) 3,4 (e) -1,4 (f) 0,2 **3.** (a) -1,1,2 (b) -3,1,3,4 (c) $\frac{4}{3}, \frac{5}{3}, 2$ (d) -1,1,2 (e) 3,7, $\frac{-1 \pm \sqrt{233}}{2}, \frac{1}{3}$

EXERCISE 7.1.4

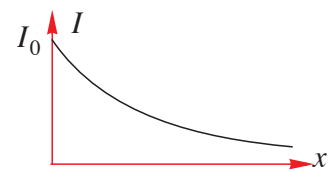
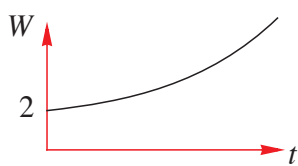
1. (a) i. 5.32 ii. 9.99 iii. 2.58 (b) i. 2.26 ii. 3.99 iii. 5.66 (c) i. 3.32 ii. -4.32 iii. -6.32 (d) i. -1.43
 ii. 1.68 iii. -2.86 **2.** (a) 0 (b) 0.54 (c) -0.21 (d) -0.75, 0 (e) 1.13 (f) 0, 0.16

EXERCISE 7.1.5

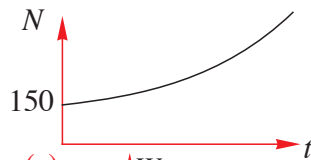
1. (a) 2 (b) -1 (c) 0.5 (d) 0.5 **2.** (a) 1 (b) 0.6 (c) 0 **3.** (a) 0 (b) $\frac{2}{3}$ **4.** (a) -1,2 (b) -2,3 (c) -1
 (d) -6,1 (e) 0,1 (f) 1 **5.** (a) 1.3863 (b) 2.1972 (c) 3.2189 (d) \emptyset **6.** (a) 0.4236 (b) 0.4055
 (c) 0.3054 (d) -0.4176 **7.** (a) 0 (b) -0.6733 (c) 0 **9.** 36 **10.** $a = \sqrt{2}e, k = \ln(\sqrt{2})$

EXERCISE 7.2

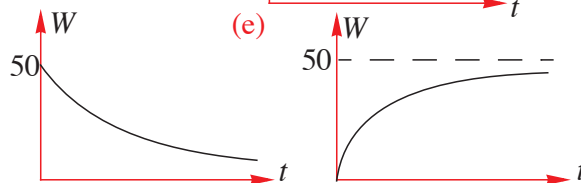
1. (a) 1000 (b) 1516 (c) 2000 (d) 10 days **2.** (a) 0.0013 (b) 2.061 kg (c) 231.56 yrs
 (d) **3.** (a) 0.01398 (b) 52.53% (c) 51.53m (d) 21.53m



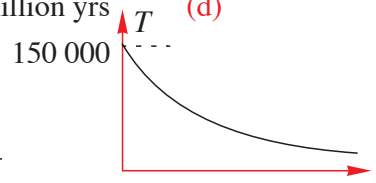
4. (a) i. 157 ii. 165 iii. 191 (b) 14.2 yrs (c) 20.1 yrs (d)



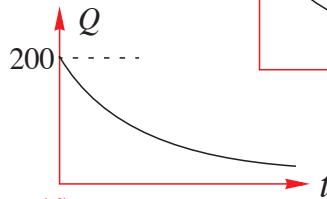
5. (a) 50 (b) 0.0222 (c) 17.99 kg (d)



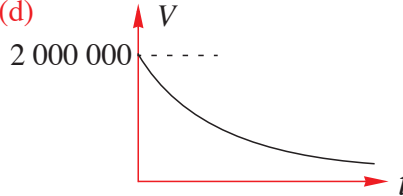
6. (a) 15 000°C (b) i. 11 900°C ii. 1500°C (c) 3.01 million yrs (d)



7. (a) 0.0151 (b) 12.50gm (c) 20 years (d)



8. (a) \$2 million (b) \$1.589 mil (c) 30.1 years (d)

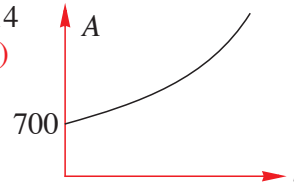


9. (b) 0.01761 (c) 199 230 (d) 22.6 years

10. (a) 20 cm² (b) 19.72 cm² (c) 100 days (d) 332 days

11. (a) 1 (b) i. 512170 ii. 517217 (c) 54.1 early 2014

12. (a) i. \$933.55 ii. \$935.50 (b) 11.95 years (c)



13. (a) 99 (b) $99 \times 2^{0.1394t}$ (c) 684

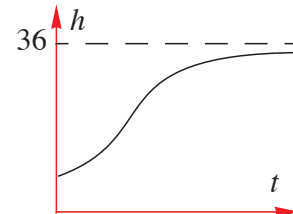
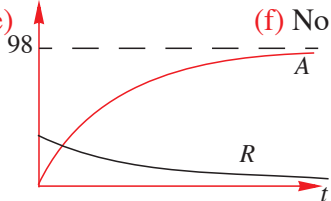
14. (a) (b) 38.85°C at ~ midnight

15. (a) 19 (b) 2.63 (c) 100

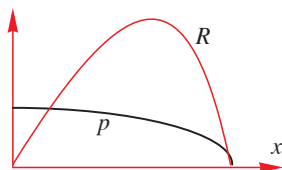
16. (a) 18 cm (b) 4 cm (c) 1.28 m (d) 36 m (e) i. 21.7yr ii. 27.6yr iii. 34.5yr (f) 36 (g)

17. (a) 5 mg/min (b) 13.51 min (c) i. 2.1, ii. 13.9 iii. 68 min

- (d) 19.6 mg (e) (f) No



18. (a) i. \$499 ii. \$496 iii. \$467 (c) 15537 (d) i. \$499k ii. \$2.48mil iii. \$4.67mil (f) 12358 (g) \$5.14mil (b), (e)



EXERCISE 7.3

1. (a) 2 (b) 2 (c) 5 (d) 3 (e) -3 (f) -2 (g) 0 (h) 0 (i) -1 (j) -2 (k) 0.5 (l) -2 2. (a) $\log_{10}10000 = 4$

- (b) $\log_{10}0.001 = -3$ (c) $\log_{10}(x+1) = y$ (d) $\log_{10}p = 7$ (e) $\log_2(x-1) = y$

- (f) $\log_2(y-2) = 4x$ 3. (a) $2^9 = x$ (b) $b^x = y$ (c) $b^{ax} = t$ (d) $10^{x^2} = z$ (e) $10^{1-x} = y$

- (f) $2^y = ax - b$ 4. (a) 16 (b) 2 (c) 2 (d) 9 (e) $\sqrt[4]{2}$ (f) 125 (g) 4 (h) 9 (i) $\sqrt[3]{\frac{1}{3}}$ (j) 21 (k) 3 (l) 13

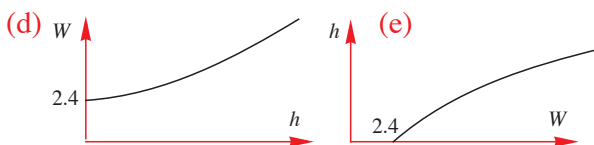
5. (a) 54.5982 (b) 1.3863 (c) 1.6487 (d) 7.3891 (e) 1.6487 (f) 0.3679 (g) 52.5982 (h) 4.7183 (i) 0.6065

EXERCISE 7.4

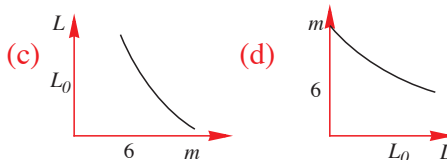
1. (a) 5 (b) 2 (c) 2 (d) 1 (e) 2 (f) 1 2. (a) $\log a = \log b + \log c$ (b) $\log a = 2\log b + \log c$
 (c) $\log a = -2\log c$ (d) $\log a = \log b + 0.5\log c$ (e) $\log a = 3\log b + 4\log c$
 (f) $\log a = 2\log b - 0.5\log c$ 3. (a) 0.18 (b) 0.045 (c) -0.09 4. (a) $x = yz$ (b) $y = x^2$
 (c) $y = \frac{x+1}{x}$ (d) $x = 2^{y+1}$ (e) $y = \sqrt{x}$ (f) $y^2 = (x+1)^3$ 5. (a) $\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{17}{15}$ (d) $\frac{3}{2}$ (e) $\frac{1}{3}$
 (f) no real sol'n (g) 3,7 (h) $\frac{\sqrt{33}-1}{2}$ (i) 4 (j) $\sqrt{10}+3$ (k) $\frac{64}{63}$ (l) $\frac{2}{15}$ 6. (a) $\log_3 2wx$ (b) $\log_{47} \frac{x}{y}$
 (c) $\log_a [x^2(x+1)^3]$ (d) $\log_a \left[\frac{(x^5)(x+1)^3}{\sqrt{2x-3}} \right]$ (e) $\log_{10} \left(\frac{y^2}{x} \right)$ (f) $\log_2 \left(\frac{y}{x} \right)$ 7. (a) 1 (b) -2 (c) 3 (d) 9
 (e) 2 (f) 9 8. (a) 1,4 (b) $1, 3^{\pm\sqrt{3}}$ (c) $1, 4^{\pm\sqrt{4}}$ (d) $1, 5^{\pm\sqrt{5}}$ 9. (a) $\frac{\log 14}{\log 2} = 3.81$ (b) $\frac{\log 8}{\log 10} = 0.90$
 (c) $\frac{\log 125}{\log 3} = 4.39$ (d) $\frac{1}{\log 2} \times \log \left(\frac{11}{3} \right) - 2 = -0.13$ (e) $\frac{\log 10 - \log 3}{4\log 3} = 0.27$ (f) 5.11
 (g) $\frac{-\log 2}{2\log 10} = -0.15$ (h) 7.37 (i) 0.93 (j) no real solution (k) $\frac{\log 3}{\log 2} - 2 = -0.42$
 (l) $\frac{\log 1.5}{\log 3} = 0.37$ 10. (a) 0.5,4 (b) 3 (c) -1,4 (d) 10,10¹⁰ (e) 5 (f) 3 11. (a) (4, $\log_4 11$)
 (b) (100,10) (c) (2,1) 12. (a) $y = xz$ (b) $y = x^3$ (c) $x = e^{y-1}$ 13. (a) $\frac{1}{e^4-1}$ (b) $\frac{1}{3}$ (c) $\frac{\sqrt{5}-1}{2}$
 (d) \emptyset 14. (a) $\ln 21 = 3.0445$ (b) $\ln 10 = 2.3026$ (c) $-\ln 7 = -1.9459$ (d) $\ln 2 = 0.6931$
 (e) $\ln 3 = 1.0986$ (f) $2\ln \left(\frac{14}{9} \right) = 0.8837$ (g) $e^3 = 20.0855$ (h) $\frac{1}{3}e^2 = 2.4630$
 (i) $\pm\sqrt{e^9} = \pm 90.0171$ (j) \emptyset (k) $e^2 - 4 = 3.3891$ (l) $\sqrt[3]{e^9} = 20.0855$ 15. (a) 0, $\ln 2$ (b) $\ln 5$
 (c) $\ln 2, \ln 3$ (d) 0 (e) 0, $\ln 5$ (f) $\ln 10$ 16. (a) 4.5222 (b) 0.2643 (c) 0,0.2619 (d) -1,0.3219
 (e) -1.2925,0.6610 (f) 0,1.8928 (g) 0.25,2 (h) 1 (i) 121.5 (j) 2

EXERCISE 7.5

1. (a) 10 (b) 30 (c) 40 2. (a) 31.64 kg (b) 1.65 (c) $W = 2.4 \times 10^{0.8h}$

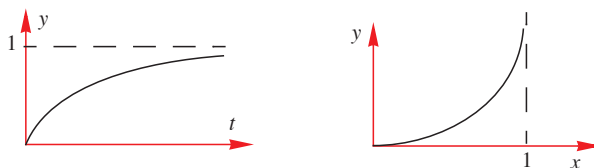


3. (a) 4.75 (b) $L = L_0 \times 10^{\left(\frac{6-m}{2.5}\right)}$



4. (a) $[0,1[$ (b) i. 2.22 ii. 1.11 iii. 0.74 yrs (c) As c increases, reliability reduces.

- (d) $x = 1 - 10^{-ct}$ (e)



5. (a) $I = \frac{a}{n^k}$ 6. (a) 0.10 (b) $\lambda = \lambda_0 \times 10^{-kx}$ (c) 16.82% (d) $k = -\frac{1}{x} \log\left(\frac{\lambda}{\lambda_0}\right)$

EXERCISE 8.1.1

1. i. (b) 4 (c) $t_n = 4n - 2$ ii. (b) -3 (c) $t_n = -3n + 23$ iii. (b) -5 (c) $t_n = -5n + 6$ iv. (b) 0.5
 (c) $t_n = 0.5n$ v. (b) 2 (c) $t_n = y + 2n - 1$ vi. (b) -2 (c) $t_n = x - 2n + 4$ 2. -28 3. 9, 17 4. -43
 5. 7 6. 7 7. -5 8. 0 9. (a) 41 (b) 31st 10. 2, $\sqrt{3}$ 11. (a) i. 2 ii. -3 (b) i. 4 ii. 11
 12. $x - 8y$ 13. $t_n = 5 + \frac{10}{3}(n - 1)$ 14. (a) -1 (b) 0

EXERCISE 8.1.2

1. (a) 145 (b) 300 (c) -170 2. (a) -18 (b) 690 (c) 70.4 3. (a) -105 (b) 507 (c) 224 4. (a) 126
 (b) 3900 (c) 14th week 5. 855 6. (a) 420 (b) -210 7. $a = 9, b = 7$

EXERCISE 8.1.3

1. 123 2. -3, -0.5, 2, 4.5, 7, 9.5, 12 3. 3.25 4. $a = 3, d = -0.05$ 5. 10 000 6. 330 7. -20
 8. 328 9. \$725, 37wks 10. i. \$55 ii. 2750 11. (a) (i) 8m (ii) 40m (b) 84m
 (c) Dist = $2n^2 - 2n = 2n(n - 1)$ (d) 8 (e) 26 players, 1300m 12. (a) 5050 (b) 10200 (c) 4233
 13. (a) 145 (b) 390 (c) -1845 14. (b) $3n - 2$

EXERCISE 8.2.1

1. (a) $r = 2, u_5 = 48, u_n = 3 \times 2^{n-1}$ (b) $r = \frac{1}{3}, u_5 = \frac{1}{27}, u_n = 3 \times \left(\frac{1}{3}\right)^{n-1}$
 (c) $r = \frac{1}{5}, u_5 = \frac{2}{625}, u_n = 2 \times \left(\frac{1}{5}\right)^{n-1}$ (d) $r = -4, u_5 = -256, u_n = -1 \times (-4)^{n-1}$
 (e) $r = \frac{1}{b}, u_5 = \frac{a}{b^3}, u_n = ab \times \left(\frac{1}{b}\right)^{n-1}$ (f) $r = \frac{b}{a}, u_5 = \frac{b^4}{a^2}, u_n = a^2 \times \left(\frac{b}{a}\right)^{n-1}$ 2. (a) ± 12
 (b) $\frac{\pm\sqrt{5}}{2}$ 3. (a) ± 96 (b) 15th 4. (a) $u_n = 10 \times \left(\frac{5}{6}\right)^{n-1}$ (b) $\frac{15625}{3888} \cong 4.02$ (c) $n = 5$ (4 times)
 5. -2, $\frac{4}{3}$ 6. (a) i. \$4096 ii. \$2097.15 (b) 6.2 yrs 7. $\left(u_n = \frac{1000}{169} \times \left(\frac{12}{5}\right)^{n-1}\right), \frac{1990656}{4225} \cong 471.16$
 8. 2.5, 5, 10 or 10, 5, 2.5 9. 53757 10. 108 952 11. (a) \$56 156 (b) \$299 284

EXERCISE 8.2.2

1. (a) 3 (b) $\frac{1}{3}$ (c) -1 (d) $-\frac{1}{3}$ (e) 1.25 (f) $-\frac{2}{3}$ 2. (a) 216513 (b) 1.6384×10^{-10} (c) $\frac{256}{729}$
 (d) $\frac{729}{2401}$ (e) $-\frac{81}{1024}$ 3. (a) 11; 354292 (b) 7; 473 (c) 8; 90.90909 (d) 8; 172.778 (e) 5; 2.256
 (f) 13; 111.1111111111 4. (a) $\frac{127}{128}$ (b) $\frac{63}{8}$ (c) $\frac{130}{81}$ (d) 60 (e) $\frac{63}{64}$ 5. 4; 118096 6. \$2109.50
 7. 9.28cm 8. (a) $V_n = V_0 \times 0.7^n$ (b) 7 9. 54 10. 53.5gms; 50 weeks. 11. 7 12. 9
 13. -0.5, -0.7797 14. $r = 5, 1.8 \times 10^{10}$ 15. \$8407.35
 16. 1.8×10^{19} or about 200 billion tonnes.

EXERCISE 8.2.3

1. Term 9 AP = 180, GP = 256. Sum to 11 terms AP = 1650, GP = 2047. 2. 18. 3. 12 4. 7, 12
 5. 8 weeks (Ken \$220 & Bo-Youn \$255) 6. (a) week 8 (b) week 12 7. (a) 1.618 (b) 121379
 [~121400, depends on rounding errors]

EXERCISE 8.2.4

1. (i) $\frac{81}{2}$ (ii) $\frac{10}{13}$ (iii) 5000 (iv) $\frac{30}{11}$ 2. $23\frac{23}{99}$ 3. 6667 fish. [Nb: $t_{43} < 1$. If we use $n = 43$ then
 ans is 6660 fish]; 20 000 fish. Overfishing means that fewer fish are caught in the long run. [An
 alternate estimate for the total catch is 1665 fish.] 4. 27 5. 48,12,3 or 16,12,9 6. (a) $\frac{11}{30}$ (b) $\frac{37}{99}$
 (c) $\frac{191}{90}$ 7. 128 cm 8. $\frac{121}{9}$ 9. $2 + \frac{4}{3}\sqrt{3}$ 10. $\frac{1 - (-t)^n}{1 + t} \frac{1}{1 + t}$ 11. $\frac{1 - (-t^2)^n}{1 + t^2} \frac{1}{1 + t^2}$

EXERCISE 8.2.5

1. 3, -0.2 2. $\frac{2560}{93}$ 3. $\frac{10}{3}$ 4. (a) $\frac{43}{18}$ (b) $\frac{458}{99}$ (c) $\frac{413}{990}$ 5. 9900 6. 3275 7. 3
 8. $t_n = 6n - 14$ 9. 6 10. $-\frac{1}{6}$ 11. i. 12 ii. 26 12. 9, 12 13. ± 2 14. (5, 5, 5), (5, -10, 20)
 15. (a) 2, 7 (b) 2, 5, 8 (c) $3n - 1$ 16. (a) 5 (b) 2 m

EXERCISE 8.3

1. \$2773.08 2. \$4377.63 3. \$1781.94 4. \$12216 5. \$35816.95 6. \$40349.37 7. \$64006.80
 8. \$276971.93, \$281325.41 9. \$63762.25 10. \$98.62, \$9467.14, interest \$4467.14.
 Flat interest = \$6000 11. \$134.41, \$3790.44, 0.602% /month (or 7.22% p.a)

EXERCISE 9.1

1.	a cm	b cm	c cm	A	B	C
1	3.8	4.1	1.6	67°	90°	23°
2	81.5	98.3	55.0	56°	90°	34°
3	32.7	47.1	33.9	44°	90°	46°
4	1.61	30.7	30.7	3°	90°	87°
5	2.3	2.74	1.49	57°	90°	33°
6	48.5	77	59.8	39°	90°	51°
7	44.4	81.6	68.4	33°	90°	57°
8	2.93	13.0	12.7	13°	90°	77°
9	74.4	94.4	58.1	52°	90°	38°
10	71.8	96.5	64.6	48°	90°	42°
11	23.3	34.1	24.9	43°	90°	47°
12	43.1	43.2	2.3	87°	90°	3°
13	71.5	80.2	36.4	63°	90°	27°
14	33.5	34.1	6.5	79°	90°	11°
15	6.1	7.2	3.82	58°	90°	32°
16	29.1	30	7.3	76°	90°	14°
17	29.0	29.1	2.0	86°	90°	4°
18	34.5	88.2	81.2	23°	90°	67°
19	24.0	29.7	17.5	54°	90°	36°
20	41.2	46.2	21.0	63°	90°	27°
21	59.6	72.9	41.8	55°	90°	35°
22	5.43	6.8	4.09	53°	90°	37°

23	13.0	19.8	14.9	41°	90°	49°
24	14.0	21.3	16.1	41°	90°	49°
25	82.4	88.9	33.3	68°	90°	22°

2. (a) $2\sqrt{3}$ (b) $5(1 + \sqrt{3})$ (c) 4 (d) $2(1 + \sqrt{3})$ (e) $\frac{4}{3}(3 + \sqrt{3})$ (f) $\sqrt{106} - 5$ 4. (a) $25(1 + \sqrt{3})$

(b) $\frac{40\sqrt{3}}{3}$

EXERCISE 9.2

1. (a) i. 030°T ii. 330°T iii. 195°T iv. 200°T (b) i. N25°E ii. S iii. S40°W iv. N10°W

2. 37.49m 3. 18.94m 4. 37° 18' 5. $\frac{26}{9}$ m/s 6. N58° 33'W, 37.23 km 7. 199.82 m 8. 10.58 m

9. 72.25 m 10. 25.39 km 11. 15.76 m 12. (a) 3.01km N, 3.99km E (b) 2.87km E 0.88km S (c) 6.86km E 2.13km N (d) 7.19km 253°T 13. 524m

EXERCISE 9.3

1. (a) 39°48' (b) 64°46' 2. (a) 12.81 cm (b) 61.35 cm (c) 77°57' (d) 60.83 cm (e) 80° 32' 3. (a) 21°48' (b) 42°2' (c) 26°34' 4. (a) 2274 (b) 12.7° 5. 251.29 m 6. (a) 103.5 m (b) 35.26°

(c) 39.23° 7. (b) 53.43 (c) 155.16 m (d) 145.68 m 8. (b) 48.54 m 9. (a) $\sqrt{(b-c)^2 + h^2}$

(b) $\tan^{-1}\left(\frac{h}{a}\right)$ (c) $\tan^{-1}\left(\frac{h}{b-c}\right)$ (d) $2(b+c)\sqrt{h^2+a^2} + 2a\sqrt{(b-c)^2+h^2}$ 10. 82.80 m

11. (a) 40.61 m (b) 49.46 m 12. (a) 10.61 cm (b) 75° 58' (c) 93° 22' 13. (a) 1.44 m (b) 73° 13' (c) 62° 11'

EXERCISE 9.4

1. (a) 1999.2 cm² (b) 756.8 cm² (c) 3854.8 cm² (d) 2704.9 cm² (e) 538.0 cm² (f) 417.5 cm² (g) 549.4 cm² (h) 14.2 cm² (i) 516.2 cm² (j) 281.5 cm² (k) 918.8 cm² (l) 387.2 cm²

(m) 139.0 cm² (n) 853.7 cm² (o) 314.6 cm² 2. 69345 m² 3. $100\pi - 6\sqrt{91}$ cm² 4. 17.34 cm

5. (a) 36.77sq units (b) 14.70 sq units (c) 62.53 sq units 6. 52.16 cm² 7. 27° 2'

8. $\frac{(b+a \times \tan \theta)^2}{2 \tan \theta}$ 9. Area of $\triangle ACD = 101.78$ cm², Area of $\triangle ABC = 61.38$ cm²

EXERCISE 9.5.1

	a cm	b cm	c cm	A	B	C
1	13.3	37.1	48.2	10°	29°	141°
2	2.7	1.2	2.8	74°	25°	81°
3	11.0	0.7	11.3	60°	3°	117°
4	31.9	39.1	51.7	38°	49°	93°
5	18.5	11.4	19.5	68°	35°	77°
6	14.6	15.0	5.3	75°	84°	21°
7	26.0	7.3	26.4	79°	16°	85°
8	21.6	10.1	28.5	39°	17°	124°
9	0.8	0.2	0.8	82°	16°	82°
10	27.7	7.4	33.3	36°	9°	135°
11	16.4	20.7	14.5	52°	84°	44°
12	21.4	45.6	64.3	11°	24°	145°
13	30.9	27.7	22.6	75°	60°	45°

14	29.3	45.6	59.1	29°	49°	102°
15	9.7	9.8	7.9	65°	67°	48°
16	21.5	36.6	54.2	16°	28°	136°
17	14.8	29.3	27.2	30°	83°	67°
18	10.5	0.7	10.9	52°	3°	125°
19	11.2	6.9	17.0	25°	15°	140°
20	25.8	18.5	40.1	30°	21°	129°

EXERCISE 9.5.2

	<i>a</i>	<i>b</i>	<i>c</i>	<i>A</i> °	<i>B</i> °	<i>C</i> °	<i>c</i> *	<i>B</i> *°	<i>C</i> *°
1	7.40	18.10	21.06	20.00	56.78	103.22	12.95	123.22	36.78
2	13.30	19.50	31.36	14.00	20.77	145.23	6.49	159.23	6.77
3	13.50	17.00	25.90	28.00	36.24	115.76	4.12	143.76	8.24
4	10.20	17.00	25.62	15.00	25.55	139.45	7.22	154.45	10.55
5	7.40	15.20	19.55	20.00	44.63	115.37	9.02	135.37	24.63
6	10.70	14.10	21.41	26.00	35.29	118.71	3.94	144.71	9.29
7	11.50	12.60	22.94	17.00	18.68	144.32	1.16	161.32	1.68
8	8.30	13.70	18.67	24.00	42.17	113.83	6.36	137.83	18.17
9	13.70	17.80	30.28	14.00	18.32	147.68	4.27	161.68	4.32
10	13.40	17.80	26.19	28.00	38.58	113.42	5.24	141.42	10.58
11	12.10	16.80	25.63	23.00	32.85	124.15	5.30	147.15	9.85
12	12.00	14.50	24.35	21.00	25.66	133.34	2.72	154.34	4.66
13	12.10	19.20	29.34	16.00	25.94	138.06	7.57	154.06	9.94
14	7.20	13.10	19.01	15.00	28.09	136.91	6.30	151.91	13.09
15	12.20	17.70	23.73	30.00	46.50	103.50	6.93	133.50	16.50
16	9.20	20.90	27.97	14.00	33.34	132.66	12.59	146.66	19.34
17	10.50	13.30	21.96	20.00	25.67	134.33	3.03	154.33	5.67
18	9.20	19.20	26.29	15.00	32.69	132.31	10.80	147.31	17.69
19	7.20	13.30	18.33	19.00	36.97	124.03	6.82	143.03	17.97
20	13.50	20.40	25.96	31.00	51.10	97.90	9.01	128.90	20.10
21	10.80	20.80	24.48	26.00	57.59	96.41	12.91	122.41	31.59
22	13.00	12.20	23.91	19.00	17.79	143.21	0.84	162.21	1.21
23	13.60	20.40	22.92	36.00	61.85	82.15	10.09	118.15	25.85
24	11.40	12.50	22.88	16.00	17.59	146.41	1.15	162.41	1.59
25	8.00	16.80	23.99	10.00	21.39	148.61	9.10	158.61	11.39
26.	(a-d) no triangles exist.								

EXERCISE 9.5.3

- 1.** 30.64 km **2.** 4.57 m **3.** 476.4 m **4.** 201°47'T **5.** 222.9 m **6.** (a) 3.40 m (b) 3.11 m
7. (b) 1.000 m (c) 1.715 m **8.** (a) 51.19 min (b) 1 hr 15.96 min (c) 14.08 km **9.** \$4886 **10.** 906 m

EXERCISE 9.5.4

	<i>a</i> cm	<i>b</i> cm	<i>c</i> cm	<i>A</i>	<i>B</i>	<i>C</i>
1	13.5	9.8	16.7	54°	36°	90°
2	8.9	10.8	15.2	35°	44°	101°
3	22.8	25.6	12.8	63°	87°	30°
4	21.1	4.4	21.0	85°	12°	83°
5	15.9	10.6	15.1	74°	40°	66°
6	8.8	13.6	20.3	20°	32°	128°
7	9.2	9.5	13.2	44°	46°	90°
8	23.4	62.5	58.4	22°	89°	69°

9	10.5	9.6	15.7	41°	37°	102°
10	21.7	36.0	36.2	35°	72°	73°
11	7.6	3.4	9.4	49°	20°	111°
12	7.2	15.2	14.3	28°	83°	69°
13	9.1	12.5	15.8	35°	52°	93°
14	14.9	11.2	16.2	63°	42°	75°
15	2.0	0.7	2.5	38°	13°	129°
16	7.6	3.7	9.0	56°	24°	100°
17	18.5	9.8	24.1	45°	22°	113°
18	20.7	16.3	13.6	87°	52°	41°
19	14.6	22.4	29.9	28°	46°	106°
20	7.0	6.6	9.9	45°	42°	93°
21	21.8	20.8	23.8	58°	54°	68°
22	1.1	1.7	1.3	41°	89°	50°
23	1.2	1.2	0.4	85°	76°	19°
24	23.7	27.2	29.7	49°	60°	71°
25	3.4	4.6	5.2	40°	60°	80°

EXERCISE 9.5.5

1. (a) 10.14 km (b) 121°T **2.** 7° 33' **3.** 4.12 cm **4.** 57.32 m **5.** 315.5 m **6.** (a) 124.3 km
(b) W28° 47' S

EXERCISE 9.5.6

1. 39.60m 52.84m **2.** 30.2m **3.** 54°, 42°, 84° **4.** 37° **5.** 028°T. **6.** 108.1cm **7.** (i) 135° (ii) 136cm
8. 41°, 56°, 83° **9.** (i) 158° left (ii) 43.22km **10.** 264m **11.** 53.33cm **12.** 186m **13.** 50.12cm
14. 5.17cm **15.** (a) 5950m (b) 13341m (c) 160° (d) 243° **17.** (a) 20.70° (b) 2.578 m (c) 1.994 m³
18. (a) 4243 m² (b) 86 m (c) 101 m

EXERCISE 9.6

1. 5.36 cm **2.** 12.3 m **3.** 24 m **4.** 40.3 m, 48.2° **5.** 16.5 min, 8.9° **6.** ~10:49 am

7. (a) i. $\frac{d \sin \phi}{\sin(\phi - \theta)}$ ii. $\frac{d \sin \theta}{\sin(\phi - \theta)}$ (b) $\frac{d \sin \phi \tan \alpha}{\sin(\phi - \theta)}$ or $\frac{d \sin \theta \tan \beta}{\sin(\phi - \theta)}$ (c) $d \left(\frac{\sin \phi \cos \theta}{\sin(\phi - \theta)} - 1 \right)$

EXERCISE 9.7

1. (i) $\frac{169\pi}{150} \text{cm}^2$, $5.2 + \frac{13\pi}{15} \text{cm}$ (ii) $\frac{529\pi}{32} \text{cm}^2$, $23 + \frac{23\pi}{8} \text{cm}$ (iii) $242\pi \text{cm}^2$, $88 + 11\pi \text{cm}$

(iv) $\frac{1156\pi}{75} \text{m}^2$, $13.6 + \frac{68\pi}{15} \text{m}$ (v) $\frac{96\pi}{625} \text{cm}^2$, $1.28 + \frac{12\pi}{25} \text{cm}$ (vi) $\frac{361\pi}{15} \text{cm}^2$, $15.2 + \frac{19\pi}{3} \text{cm}$

(vii) $5248.8\pi \text{m}^2$, $648 + 32.4\pi \text{cm}$ (viii) $\frac{12943\pi}{300} \text{cm}^2$, $17.2 + \frac{301\pi}{30} \text{cm}$

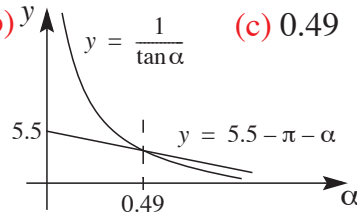
(ix) $\frac{1922\pi}{75} \text{cm}^2$, $12.4 + \frac{124\pi}{15} \text{cm}$ (x) $\frac{15884\pi}{3} \text{cm}^2$, $152 + \frac{418\pi}{3} \text{cm}$ (xi) $12\pi \text{cm}^2$, $24 + 2\pi \text{cm}$

(xii) $\frac{98\pi}{3} \text{cm}^2$, $28 + \frac{14\pi}{3} \text{cm}$ (xiii) $\frac{196\pi}{75} \text{cm}^2$, $5.6 + \frac{28\pi}{15} \text{cm}$ (xiv) $\frac{11532\pi}{25} \text{cm}^2$, $49.6 + \frac{186\pi}{5} \text{cm}$

(xv) $\frac{3\pi}{50} \text{cm}^2$, $2.4 + \frac{\pi}{10} \text{cm}$ **2.** 0.63^c , 36° **3.** 0.0942m^3 **4.** 1.64^c **5.** 79cm. **6.** 5.25cm^2

7. (a) 31.83m (b) 406.28m (c) 11° **8.** 1.11^c **9.** 0.75^c **10.** (a) 1.85^c (b) i. 37.09 cm ii. 88.57 cm
(c) 370.92cm^2 **11.** 26.57cm^2 **12.** 193.5 cm **13.** (a) 105.22 cm (b) 118.83 cm **14.** (a) 9 cm

(b) 12 cm (c) $36^\circ 52'$ **15.** (b) $y = \frac{1}{\tan \alpha}$ (c) 0.49 **16.** 1439.16 cm²



EXERCISE 10.1

- 1.** (a) 120° (b) 108° (c) 216° (d) 50° **2.** (a) π^c (b) $\frac{3\pi^c}{2}$ (c) $\frac{7\pi^c}{9}$ (d) $\frac{16\pi^c}{9}$ **3.** (a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{2}$
(c) $-\sqrt{3}$ (d) -2 (e) $-\frac{1}{2}$ (f) $-\frac{\sqrt{3}}{2}$ (g) $\frac{1}{\sqrt{3}}$ (h) $\sqrt{3}$ (i) $-\frac{1}{\sqrt{2}}$ (j) $-\frac{1}{\sqrt{2}}$ (k) 1 (l) $-\sqrt{2}$ (m) $-\frac{1}{\sqrt{2}}$ (n) $\frac{1}{\sqrt{2}}$
(o) -1 (p) $\sqrt{2}$ (q) 0 (r) 1 (s) 0 (t) undefined **4.** (a) 0 (b) -1 (c) 0 (d) -1 (e) $\frac{1}{\sqrt{2}}$ (f) $-\frac{1}{\sqrt{2}}$ (g) -1
(h) $\sqrt{2}$ (i) $-\frac{1}{2}$ (j) $-\frac{\sqrt{3}}{2}$ (k) $\frac{1}{\sqrt{3}}$ (l) $\sqrt{3}$ (m) $-\frac{\sqrt{3}}{2}$ (n) $\frac{1}{2}$ (o) $-\sqrt{3}$ (p) 2 (q) $-\frac{1}{\sqrt{2}}$ (r) $\frac{1}{\sqrt{2}}$ (s) -1
(t) $-\sqrt{2}$ **5.** (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{1}{2}$ (e) $-\frac{1}{\sqrt{3}}$ (f) $-\frac{1}{2}$ (g) $-\sqrt{2}$ (h) $-\frac{2}{\sqrt{3}}$ **6.** (a) $-\frac{1}{2}$ (b) $-\frac{1}{\sqrt{2}}$
(c) $\sqrt{3}$ (d) -2 (e) 1 (f) $\frac{1}{2}$ (g) $-\frac{1}{\sqrt{3}}$ (h) $-\frac{\sqrt{3}}{2}$ (i) $-\frac{2}{\sqrt{3}}$ (j) $\frac{1}{\sqrt{3}}$ (k) $\frac{2}{\sqrt{3}}$ (l) $-\frac{\sqrt{3}}{2}$ **7.** (a) $(\frac{1}{2}, \frac{\sqrt{3}}{2})$
(b) $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ (c) $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ (d) $(\frac{\sqrt{3}}{2}, -\frac{1}{2})$ **8.** (a) 0 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ **10.** (a) $-\frac{2}{3}$
(b) $-\frac{2}{3}$ (c) $-\frac{2}{3}$ **11.** (a) $-\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{2}{5}$ **12.** (a) k (b) $-\frac{1}{k}$ (c) $-k$ **13.** (a) $\frac{\sqrt{5}}{3}$ (b) $\frac{3}{\sqrt{5}}$ (c) $-\frac{\sqrt{5}}{3}$
14. (a) $-\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ **15.** (a) $\frac{4}{5}$ (b) $\frac{3}{4}$ (c) $-\frac{5}{3}$ **16.** (a) $-k$ (b) $-\sqrt{1-k^2}$ (c) $-\frac{k}{\sqrt{1-k^2}}$
17. (a) $-\sqrt{1-k^2}$ (b) $\frac{k}{\sqrt{1-k^2}}$ (c) $-\frac{1}{\sqrt{1-k^2}}$ **18.** (a) $\sin \theta$ (b) $\cot \theta$ (c) 1 (d) 1 (e) $\cot \theta$ (f) $\tan \theta$
19. (a) $\frac{\pi}{3}, \frac{2\pi}{3}$ (b) $\frac{\pi}{3}, \frac{5\pi}{3}$ (c) $\frac{\pi}{3}, \frac{4\pi}{3}$ (d) $\frac{5\pi}{6}, \frac{7\pi}{6}$ (e) $\frac{5\pi}{6}, \frac{11\pi}{6}$ (f) $\frac{7\pi}{6}, \frac{11\pi}{6}$

EXERCISE 10.2.1

- 3.** (a) $x^2 + y^2 = k^2, -k \leq x \leq k$ (b) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, -b \leq x \leq b$ (c) $(x-1)^2 + (2-y)^2 = 1, 0 \leq x \leq 2$
(d) $\frac{(1-x)^2}{b^2} + \frac{(y-2)^2}{a^2} = 1$ (e) $5x^2 + 5y^2 + 6xy = 16$ **4.** (a) (i) $-\frac{4}{5}$ (ii) $-\frac{5}{3}$ (b) (i) $\frac{4}{\sqrt{7}}$ (ii) $-\frac{\sqrt{7}}{3}$
5. (a) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ (b) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (c) 0, $\frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi$ (d) $\frac{\pi}{2}, \frac{3\pi}{2}$ **9.** (a) $\frac{2a}{a^2+1}$ (b) $\frac{a^2-1}{a^2+1}$
10. (a) i. 1 ii. 1 (b) 1 **11.** (a) $\frac{1-\sqrt{x^2-1}}{x}$ (b) $\frac{1+\sqrt{x^2-1}}{x}$ (c) $\frac{2}{x^2}-1$ **12.** (a) i. 6 ii. $\frac{5}{2}$ iii. $\frac{9}{8}$
(b) i. 5 ii. 1 iii. -2 **13.** (a) ± 2 (b) $\frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}$ or $\frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}$

14. (a) i. $\frac{1}{5^4}$ ii. $\frac{1}{3}$ (b) i. 27 ii. $\frac{1}{3}$ **15.** (a) $1 + 2k$ (b) $(1 - k)\sqrt{1 + 2k}$

16. (a) $\frac{1-a}{2\sqrt{a}}$ (b) i. $2 + \sqrt{2a - a^2}$ ii. $\frac{-\sqrt{2a - a^2}}{1 - a}$ **17.** (a) $\frac{2}{3}$ (b) $0, \pm \frac{2\sqrt{2}}{3}$ **18.** $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

EXERCISE 10.2.2

1. (a) $\sin \alpha \cos \phi + \cos \alpha \sin \phi$ (b) $\cos 3\alpha \cos 2\beta - \sin 3\alpha \sin 2\beta$ (c) $\sin 2x \cos y - \cos 2x \sin y$

(d) $\cos \phi \cos 2\alpha + \sin \phi \sin 2\alpha$ (e) $\frac{\tan 2\theta - \tan \alpha}{1 + \tan 2\theta \tan \alpha}$ (f) $\frac{\tan \phi - \tan 3\omega}{1 + \tan \phi \tan 3\omega}$ **2.** (a) $\sin(2\alpha - 3\beta)$

(b) $\cos(2\alpha + 5\beta)$ (c) $\sin(x + 2y)$ (d) $\cos(x - 3y)$ (e) $\tan(2\alpha - \beta)$ (f) $\tan x$ (g) $\tan\left(\frac{\pi}{4} - \phi\right)$

(h) $\sin\left(\frac{\pi}{4} + \alpha + \beta\right)$ (i) $\sin 2x$ **3.** (a) $-\frac{56}{65}$ (b) $\frac{33}{65}$ (c) $-\frac{16}{63}$ **4.** (a) $\frac{16}{65}$ (b) $\frac{63}{65}$ (c) $\frac{56}{33}$

5. (a) $-\frac{5\sqrt{11}}{18}$ (b) $-\frac{7}{18}$ (c) $\frac{5\sqrt{11}}{7}$ (d) $\frac{35\sqrt{11}}{162}$ **6.** (a) $-\frac{3}{5}$ (b) $-\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{24}{7}$ **7.** (a) $\frac{1 + \sqrt{3}}{2\sqrt{2}}$

(b) $\frac{1 + \sqrt{3}}{2\sqrt{2}}$ (c) $-\frac{1 + \sqrt{3}}{2\sqrt{2}}$ (d) $\sqrt{3} - 2$ **8.** (a) $\frac{2ab}{a^2 + b^2}$ (b) $\frac{a^2 + b^2}{2ab}$ (c) $\frac{a^4 - 6a^2b^2 + b^4}{(a^2 + b^2)^2}$ (d) $\frac{2ab}{b^2 - a^2}$

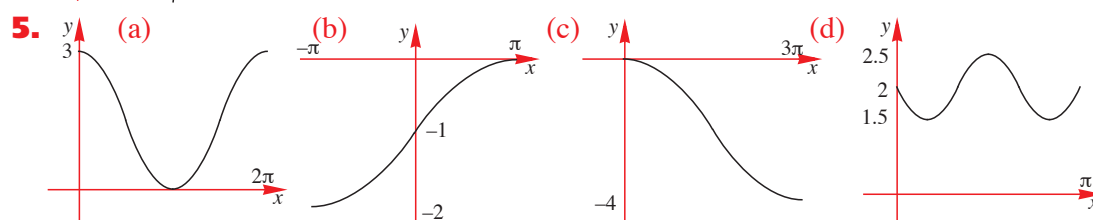
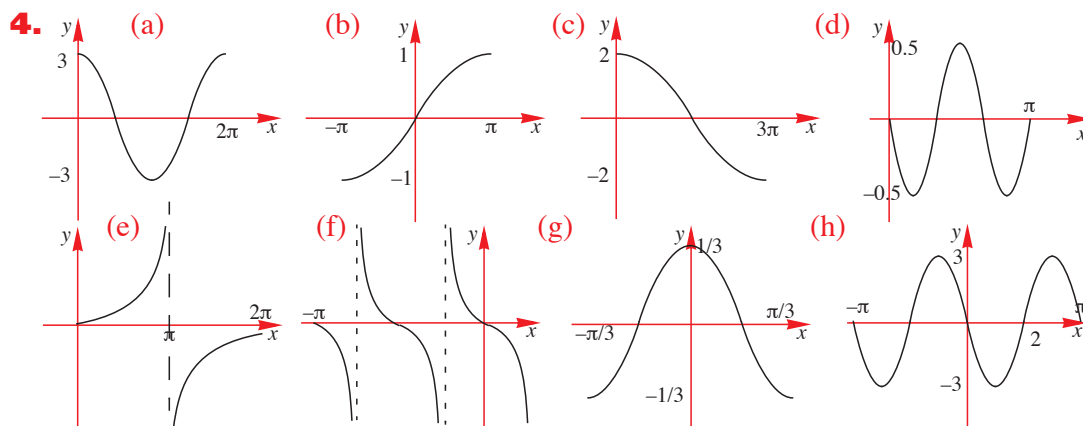
12. $\sqrt{2} - 1$ **14.** (a) $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$ (b) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ (c) $0, \pi, 2\pi, \alpha, \pi \pm \alpha, 2\pi - \alpha, \alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

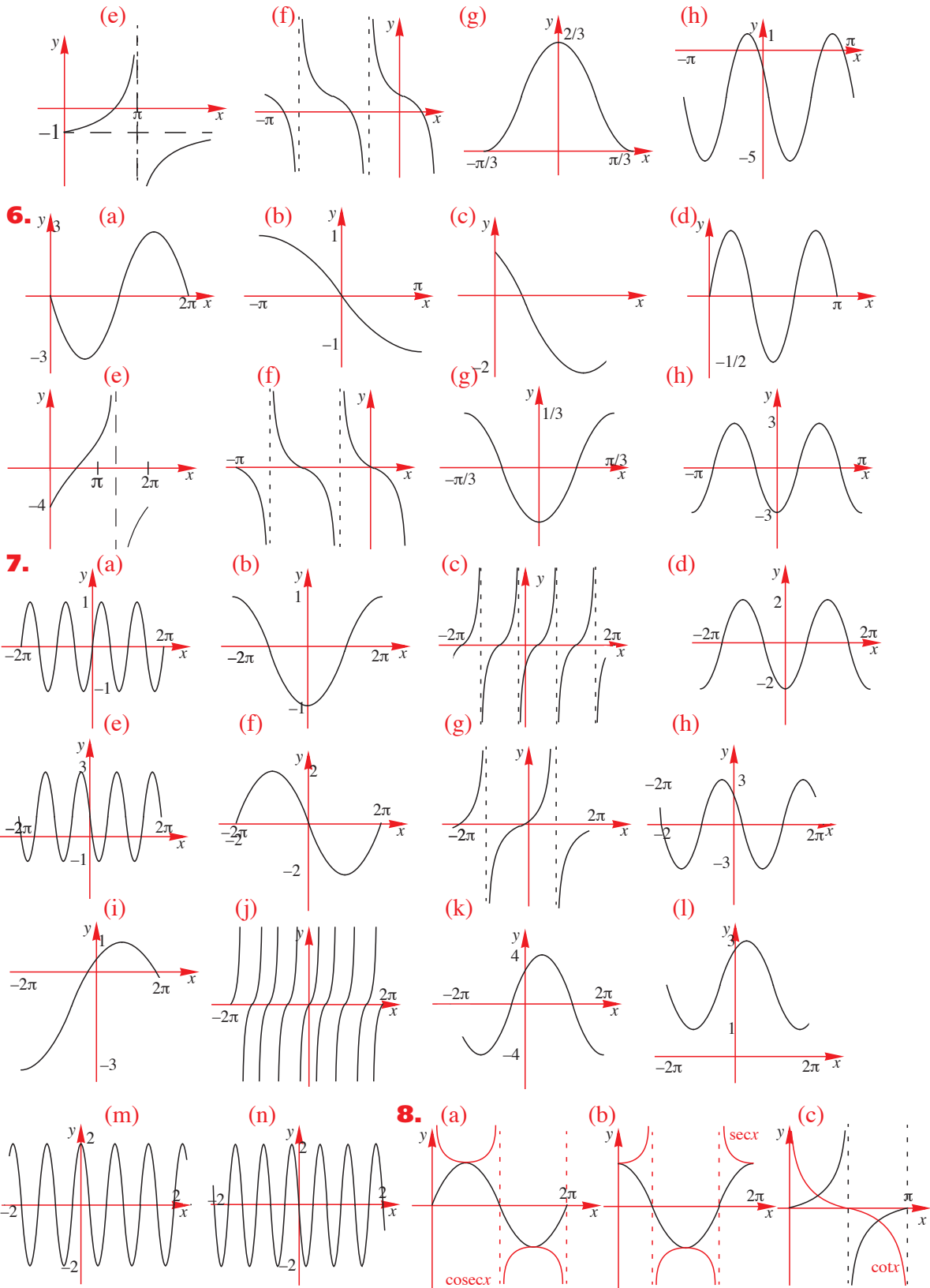
15. (a) $R = \sqrt{a^2 + b^2}, \tan \alpha = \frac{b}{a}$ (b) 10 **16.** (a) $R = \sqrt{a^2 + b^2}, \tan \alpha = \frac{b}{a}$ (b) -11 **18.** $2 - \sqrt{3}$

EXERCISE 10.3

1. (a) 4π (b) $\frac{2\pi}{3}$ (c) 3π (d) 4π (e) 2 (f) $\frac{\pi}{2}$ **2.** (a) 5 (b) 3 (c) 5 (d) 0.5 **3.** (a) $2\pi, 2$ (b) $6\pi, 3$ (c) π

(d) π (e) $\pi, 4$ (f) $\pi, 3$ (g) 6π (h) $\frac{2\pi}{3}, \frac{1}{4}$ (i) 3π (j) $\frac{8\pi}{3}, \frac{2}{3}$



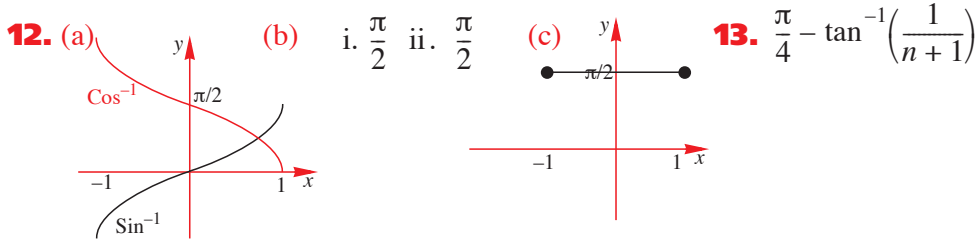
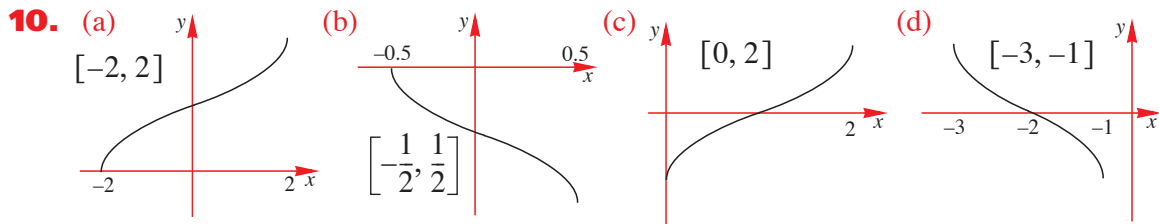


EXERCISE 10.4

1. (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{\pi}{3}$ (e) $\frac{\pi}{4}$ (f) $-\frac{\pi}{3}$ (g) 1.1071° (h) -0.7754° (i) 0.0997° (j) 1.2661°
 (k) -0.6435° (l) 1.3734° (m) undefined (n) -1.5375° (o) 1.0654° 2. (a) -1 (b) $\frac{\sqrt{3}}{4}$ (c) $-\frac{1}{3\sqrt{2}}$

4. $\frac{1}{3}, \frac{1}{2}$ 5. (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ (e) $\frac{3\sqrt{2}}{4}$ (f) -1 6. (a) 1 (b) $-\frac{7}{25}$ (c) $\frac{63}{65}$ (d) undefined

(e) $\frac{4\sqrt{5}}{9}$ (f) $\frac{3}{5}$ (g) $\frac{4}{3}$ (h) $\frac{1}{2}$ 9. (a) $\frac{\sqrt{1-k^2}}{k}$ (b) $\frac{1}{\sqrt{1+k^2}}$



EXERCISE 10.5

1. (a) $\frac{\pi}{4}, \frac{3\pi}{4}$ (b) $\frac{7\pi}{6}, \frac{11\pi}{6}$ (c) $\frac{\pi}{3}, \frac{2\pi}{3}$ (d) $\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$ (e) $\frac{\pi}{3}, \frac{5\pi}{3}$

(f) $\frac{5}{4}, \frac{7}{4}, \frac{13}{4}, \frac{15}{4}, \frac{21}{4}, \frac{23}{4}$ 2. (a) $\frac{\pi}{4}, \frac{7\pi}{4}$ (b) $\frac{2\pi}{3}, \frac{4\pi}{3}$ (c) $\frac{\pi}{6}, \frac{11\pi}{6}$ (d) π (e) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (f) $\frac{3}{2}, \frac{5}{2}, \frac{11}{2}$

3. (a) $\frac{\pi}{6}, \frac{7\pi}{6}$ (b) $\frac{3\pi}{4}, \frac{7\pi}{4}$ (c) $\frac{\pi}{3}, \frac{4\pi}{3}$ (d) $4\tan^{-1}2$ (e) $\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$ (f) 3 4. (a) $90^\circ, 330^\circ$

(b) $180^\circ, 240^\circ$ (c) $90^\circ, 270^\circ$ (d) $65^\circ, 335^\circ$ (e) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ (f) $0, \pi, 2\pi$ (g) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(h) $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$ 5. (a) $60^\circ, 300^\circ$ (b) $\frac{4\pi}{3}, \frac{5\pi}{3}$ (c) $\frac{\pi}{6}, \frac{7\pi}{6}$ (d) $23^\circ 35', 156^\circ 25'$ (e) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(f) $\frac{2\pi}{3}, \frac{5\pi}{3}$ (g) $\frac{5\pi}{6}, \frac{9\pi}{6}$ (h) $3.3559^\circ, 5.2105^\circ$ (i) $\frac{\pi}{3}, \frac{4\pi}{3}$ (j) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ (k) $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

(l) $68^\circ 12', 248^\circ 12'$ (m) $\frac{\pi}{3}, \frac{5\pi}{3}$ (n) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ (o) \emptyset 6. (a) $-\frac{3\pi}{4}, \frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $-\frac{7\pi}{8}, -\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}$

(d) $-\frac{\pi}{2}$ (e) $\pm\frac{\pi}{2}$ (f) $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$ (g) $\frac{\pi}{2}, \frac{3\pi}{2}$ (h) $\frac{\pi}{2}, \frac{3\pi}{2}$ 7. (a) $\frac{3\pi}{4}, \frac{7\pi}{4}, \tan^{-1}\left(\frac{2}{3}\right), \pi + \tan^{-1}\left(\frac{2}{3}\right)$

(b) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{4}$ 8. (a) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$ (b) $\frac{2\pi}{3}, \frac{4\pi}{3}$

(c) $0, 1, 2, 3, 4, 5, 6$ 9. (a) $\frac{\pi}{3}, \frac{5\pi}{3}, \pi \pm \cos^{-1}\left(\frac{1}{4}\right)$ (b) $\frac{3\pi}{4}, \frac{7\pi}{4}, \tan^{-1}(3), \pi + \tan^{-1}(3)$

(c) $\frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$ (d) $\tan^{-1}\left(\frac{3}{2}\right), \pi - \tan^{-1}(2), \pi + \tan^{-1}\left(\frac{3}{2}\right), 2\pi - \tan^{-1}(2)$ 10. (a) $2\sin\left(x + \frac{\pi}{6}\right)$

(b) $0, \frac{2\pi}{3}, 2\pi$ 11. (a) $2\sin\left(x - \frac{\pi}{3}\right)$ (b) $\frac{\pi}{6}, \frac{3\pi}{2}$ 12. $\frac{\pi}{3}, \frac{2\pi}{3}$ 13. i. $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right) \cup \left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$

ii. $\left(\pi + \sin^{-1}\left(\frac{1}{\sqrt{3}}\right), 2\pi - \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) \cup \left(3\pi + \sin^{-1}\left(\frac{1}{\sqrt{3}}\right), 4\pi - \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$

14. (a) ii. $\left[0, \frac{\pi}{4}\right] \cup \left(\frac{5\pi}{4}, 2\pi\right]$ (b) ii. $\left[0, \frac{\pi}{6}\right] \cup \left(\frac{\pi}{2}, \frac{5\pi}{6}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$

16. (a) i. $\{x|x = k\pi + \alpha(-1)^k, k \in \mathbb{Z}\}$ ii. $\{x|2k\pi + \alpha \leq x \leq (2k+1)\pi - \alpha, k \in \mathbb{Z}\}$

(b) $\left\{x|x = (2k+1)\frac{\pi}{5}\right\} \cup \{x|x = 2k\pi\}, k \in \mathbb{Z}$

(c) $\left\{x|x = \frac{2k\pi}{5} + \frac{\pi}{10}\right\} \cup \left\{x|x = 2k\pi - \frac{\pi}{2}\right\}, k \in \mathbb{Z}$ **17.** (a) $0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$ (b) $\sqrt{2}, \frac{\sqrt{2}}{2}$

18. (c) $2\cos\frac{\pi}{9}, 2\cos\frac{5\pi}{9}, 2\cos\frac{7\pi}{9}$ **19.** $\left\{\pm\frac{\pi}{4}, \pm\frac{2\pi}{3}, \pm\frac{3\pi}{4}\right\}$ **21.** (a) $90^\circ, 199^\circ 28', 340^\circ 32'$

(b) $(199^\circ 28', 340^\circ 32')$ **24.** $\left\{(x, y)|x = 2k\pi + \frac{\pi}{2}, y = 2k\pi\right\} \cup \left\{(x, y)|x = 2k\pi - \frac{\pi}{2}, y = 2k\pi + \pi\right\}, k \in \mathbb{Z}$

EXERCISE 10.6

1. (a) 5, 24, 11, 19 (b) $T = 5\sin\left(\frac{\pi t}{12} - 3\right) + 19$ (c) 23.6° **2.** (a) 3, 4.2, 2, 7

(b) $L = 3\sin\left(\frac{\pi t}{2.1} - 3\right) + 7$ **3.** (a) 5, 11, 0, 7 (b) $V = 5\sin\left(\frac{2\pi t}{11}\right) + 7$ **4.** (a) 1, 11, 1, 12

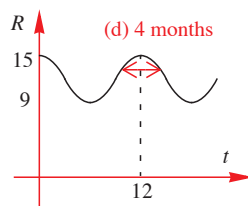
(b) $P = \sin\frac{2\pi}{11}(t-1) + 12$ **5.** (a) 2.6, 7, 2, 6 (b) $S = 2.6\sin\frac{2\pi}{7}(t-2) + 6$ **6.** (a) 0.6, 3.5, 0, 11

(b) $P = 0.6\sin\left(\frac{4\pi t}{7}\right) + 11$ **7.** (a) 0.8, 4.6, 2.7, 11 (b) $D = 0.8\sin\frac{\pi}{2.3}(t-2.7) + 11$ **8.** (a) 3000

(b) 1000, 5000 (c) $\frac{4}{9}$ **9.** (a) 6.5 m, 7.5 m (b) 1.58 sec, 3.42 sec **10.** (a) 750, 1850 (b) 3.44

(c) Mid-April to End of August

11. (a) 15000 (b) 12 months (c) (d) 4 months

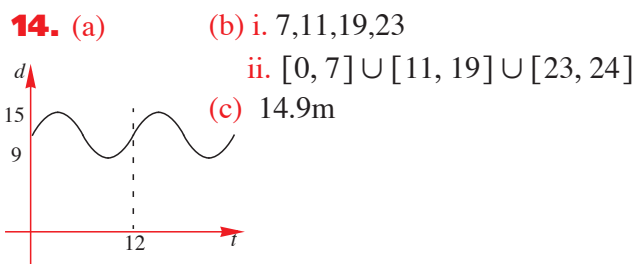
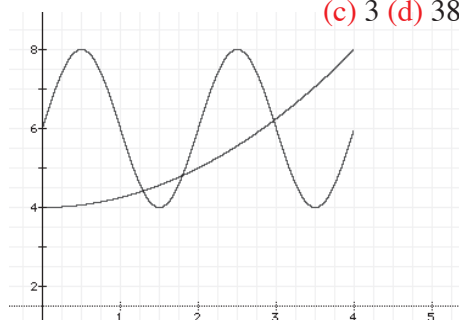


12. (a) $\pi, -2, 2$ (b) $\frac{1}{3}$ m (c) $\frac{4}{3}$ m

13.(a)

t	0	0.5	1	1.5	2	2.5	3	3.5	4
$F(t)$	6	8	6	4	6	8	6	4	6
$G(t)$	4	4.0625	4.25	4.5625	5	5.5625	6.25	7.0625	8

(b) **(c)** 3 **(d)** 38.45%



EXERCISE 11.1

- 1.** a. i. 2 ii. -3 iii. 6 iv. 0 v. $\frac{3}{2}$ vi. $\frac{1}{3}$ b. i. 2 ii. $\sqrt{2}$ iii. -5 iv. $-\frac{2}{5}$ v. $\frac{1}{2}$ vi. -1 c. i. $2 - 2i$ ii. $-3 - \sqrt{2}i$
iii. $6 + 5i$ iv. $\frac{2}{5}i$ v. $\frac{3}{2} - \frac{1}{2}i$ vi. $\frac{1}{3} + i$ **2.** a. $7 + i$ b. $1 - 3i$ c. $15 - 8i$ d. $-1 - 8i$ e. $10 + 11i$
f. $-2 + 3i$ **3.** a. $-1 + 3i$ b. $5 - i$ c. $-4 + 3i$ d. $6i$ e. $-4 + 7i$ f. $-2 + 3i$ **4.** a. $\frac{1}{2}(1 + i)$
b. $-\frac{1}{2}(5 + i)$ c. $-1 - 2i$ d. $\frac{1}{2}i$ e. 1 f. $-\frac{1}{13}(5 + i)$ **5.** a. $14 + 8i$ b. $-2 - 2i$ c. $-2\sqrt{2} - i$
d. $\frac{1}{5}(2 + i)$ e. $2 - i$ f. $\frac{1}{5}(1 + 3i)$ **6.** a. $\frac{1}{2}$ b. $\frac{1}{2}(3 + \sqrt{2})$ c. $3 + \sqrt{2}$ **7.** a. $x = 4, y = -\frac{1}{2}$
b. $x = -5, y = 12$ c. $x = 0, y = 5$ **8.** a. i. $1, i, -1, -i, 1, i$ ii. $-i, -1, i, 1, -i$ b. i. -1 ii. $-i$
iii. -1 iv. -1 **9.** $x = -\frac{120}{29}, y = \frac{39}{29}$ **12.** a. $x = 0$ or $y = 0$ or both b. $x^2 - y^2 = 1$ **13.** a. $3 - i$
b. $2 - i$ **14.** a. $4i$ b. -4 c. $-i$ **15.** a. $x = 13, y = 4$ b. $x = 4, y = \frac{4}{3}$ **16.** **17.** $-\frac{1}{3}(1 + 2\sqrt{2}i)$
18. $(u, v) = \left(\frac{1}{2}(\sqrt{2} + 2), \frac{1}{2}\sqrt{2}\right), \left(\frac{1}{2}(2 - \sqrt{2}), -\frac{1}{2}\sqrt{2}\right)$ **19.** (a) $-\frac{7}{2}$ (b) $-\frac{1}{5}$ **21.** $\pm \frac{\sqrt{2}(1 + i)}{2}$
22. (a) $\cos(\theta + \alpha) + i\sin(\theta + \alpha)$ (b) $\cos(\theta - \alpha) + i\sin(\theta - \alpha)$
(c) $r_1 r_2 (\cos(\theta + \alpha) + i\sin(\theta + \alpha))$ (d) $x^2 - 2x\cos(\theta) + 1$ (e) $x^2 + 2x\sin(\alpha) + 1$ **24.** (a) $3 + i$
(b) 325 (c) $(x^2 + y^2)^2$ **25.** $z = 4, b = -4$ **26.** (a) $\cos(\theta) + i\sin(\theta)$ (b) $\cos(4\theta) + i\sin(4\theta)$
27. (a) $\begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\beta^2 \end{bmatrix}$ (b) $\begin{bmatrix} \alpha^4 & 0 \\ 0 & \beta^4 \end{bmatrix}$ (c) $\begin{bmatrix} -\frac{i}{\alpha} & 0 \\ 0 & \frac{i}{\beta} \end{bmatrix}$ (d) $\begin{bmatrix} \alpha^{4n} & 0 \\ 0 & \beta^{4n} \end{bmatrix}$ **28.** (a) $-\sin(\theta) + i\cos(\theta)$
(d) $\cos(\theta) - i\sin(\theta)$

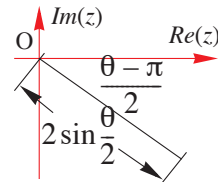
EXERCISE 11.2

- 1.** The points to plot are: $(2, 1), (0, -6), (4, -3), (2, -2), (-3, 3), (-3, 4), (0, -0.5), (1, -1)$. **2.** (a) i. $i - 1$
ii. $-(1 + i)$ iii. $1 - i$ iv. $1 + i$; Anticlockwise rotation of 90° . (b) i. Reflection about the $Re(z)$
axis. ii. Results will always be a real number, so the point will always lie on $Re(z)$ axis. iii. Point
will always lie on the $Im(z)$ axis. **3.** (a) $-3 + 4i$ (b) $\frac{1}{2}(1 - i)$ (c) $-1 + 3i$ (d) $1 + 3i$
(e) $-1 - 3i$ (f) $2 - 3i$ (g) $\frac{1}{2}(3 + i)$ (h) $\frac{1}{5}(3 - i)$ **4.** (a) $2; \frac{\pi}{3}$ (b) $2; -\frac{\pi}{3}$ (c) $\sqrt{3}; \arctan(\sqrt{2})$
(d) $3; \frac{\pi}{2}$ (e) $1; -\frac{2\pi}{3}$ (f) $1; \frac{\pi}{4}$ (g) $6; 0$ (h) $\frac{5}{4}; -\arctan\left(\frac{4}{3}\right)$ **5.** (a) $\sqrt{a^2 + b^2}; \sqrt{a^2 + b^2}; a^2 + b^2$
(b) i. $\frac{\pi}{2}$ or $-\frac{\pi}{2}$ ii. 0 or π **6.** i. $\sqrt{2x^2 + 18}$ ii. ± 3 **7.** Triangle property; the sum of the lengths of
two sides of a triangle is larger than the third side. **8.** 0 **9.** (a) 15 (b) 5 (c) 10 **12.** (b) $1 + i$
14. (a) $5; -53.13^\circ$ (b) $\sqrt{2}; -45^\circ$ (c) $1; -90^\circ$ **15.** $\frac{\pi}{4}, \pi$ **16.** (a) $1; \theta$ (b) $1; \frac{\pi}{2} - \theta$ (c) $1; -\theta$
17. (a) $|\sec\alpha|, \alpha - \pi$ (for Principal argument) otherwise, $\alpha + k\pi$. where k is an integer.

(b) $|\sec \alpha|$, $\alpha + \frac{\pi}{2}$ (for Principal argument) otherwise, $\left(\alpha + \frac{\pi}{2}\right) + k\pi$, k is an integer.

(c) $2 \left| \cos\left(\frac{\theta}{2}\right) \right|$, $\frac{\theta}{2}$ (for Principal argument) otherwise, $\frac{\theta}{2} + k\pi$, k is an integer.

18. (a) i. $\frac{\sqrt{3}}{2} + \frac{i}{2}$ ii. $\frac{\pi}{6}$ (b) $-\frac{5\pi}{6}$ 19. (a) i. $2 \sin \frac{\theta}{2}$ ii. $\frac{\theta - \pi}{2}$ (b)



EXERCISE 11.3

1. (a) $\sqrt{2} \operatorname{cis}\left(\frac{\pi}{4}\right)$ (b) $\sqrt{2} \operatorname{cis}\left(\frac{3\pi}{4}\right)$ (c) $\sqrt{2} \operatorname{cis}\left(-\frac{3\pi}{4}\right)$ 2. (a) $2\sqrt{2} \operatorname{cis}\left(\frac{\pi}{4}\right)$ (b) $2 \operatorname{cis}\left(\frac{\pi}{6}\right)$

(c) $4\sqrt{2} \operatorname{cis}\left(-\frac{\pi}{4}\right)$ (d) $5 \operatorname{cis}(53^\circ 7')$ (e) $\sqrt{5} \operatorname{cis}(153^\circ 26')$ (f) $\sqrt{13} \operatorname{cis}(-123^\circ 41')$ (g) $2 \operatorname{cis}\left(\frac{5\pi}{6}\right)$

(h) $\operatorname{cis}\left(-\frac{\pi}{3}\right)$ (i) $\sqrt{10} \operatorname{cis}(-18^\circ 26')$ 3. (a) $2i$ (b) $\frac{3\sqrt{3}}{2} + \frac{3}{2}i$ (c) $1 - i$ (d) $-5i$ (e) $-4 + 4\sqrt{3}i$

(f) $\frac{1}{6}(\sqrt{2} + \sqrt{6}i)$ 4. (a) $\sqrt{\frac{5}{3}}$ (b) 1 (c) 0 5. (a) $1 - \sqrt{3}i$ (b) $1 - i$ (c) $(1 - \sqrt{3}) + (1 + \sqrt{3})i$

7. (a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) $\frac{\pi}{4}$ (e) $\frac{2\pi}{3}$ (f) $\frac{11\pi}{12}$ 8. (a) $-4(1 + i)$ (b) -4 (c) $-16 + 16i$

(d) $-8 - 8\sqrt{3}i$ (e) $-16\sqrt{3} - 16i$ (f) $-117 - 44i$ 9. (a) $\frac{1}{8}(-1 + i)$ (b) $-\frac{1}{4}$ (c) $-\frac{1}{32}(1 + i)$

(d) $\frac{1}{32}(-1 + \sqrt{3}i)$ (e) $\frac{1}{64}(-\sqrt{3} + i)$ (f) $\frac{1}{15625}(-117 + 44i)$ 10. (a) $-8i$ (b) $\frac{81}{2}(-1 + \sqrt{3}i)$

(c) $\frac{1}{2}i$ (d) $-\frac{1}{125}i$ (e) $-\frac{1}{16}(1 + \sqrt{3}i)$ (f) $-\frac{2}{81}(1 + \sqrt{3}i)$ 11. (a) $128(1 - i)$ (b) $4\sqrt{3} - 4i$ (c) $-32i$

(d) 256 (e) $\frac{11753}{625} - \frac{10296}{625}i$ (f) $-2i$ 12. (b) i. -1 ii. -1 iii. i 13. (a) $-i$ (b) $6\sqrt{2}(1 + i)$

(c) $-\sqrt{2 - \sqrt{2}} + \sqrt{2 + \sqrt{2}}i$ 14. (a) $\frac{\sqrt{2}}{2}(1 + i)$; $\frac{1}{2}(1 + \sqrt{3}i)$ $\frac{\sqrt{2}}{4}((1 - \sqrt{3}) + (1 + \sqrt{3})i)$

(b) i. $\frac{\sqrt{2}}{4}(1 + \sqrt{3})$ ii. $\frac{\sqrt{2}}{4}(1 - \sqrt{3})$ 20. (c) 3 22. $(\operatorname{cis}(-\theta))^3$

23. $(\cos 2\theta + \cos 2\alpha)(\cos(\theta - \alpha) - i \sin(\theta - \alpha))$ [or $2 \cos(\alpha - \theta)$] 25. (a) $\operatorname{cosec} \theta$ (b) $\theta - \frac{\pi}{2}$

EXERCISE 11.4.1

1. (a) $(x - 3 + i)(x - 3 - i)$ (b) $(x + 2 + 3i)(x + 2 - 3i)$ (c) $(x - 1 + i)(x - 1 - i)$

(d) $(z + 2 + i)(z + 2 - i)$ (e) $\left(z - \frac{(3 + \sqrt{7}i)}{2}\right)\left(z - \frac{(3 - \sqrt{7}i)}{2}\right)$ (f) $(z + 5 + \sqrt{5}i)(z + 5 - \sqrt{5}i)$

(g) $4\left(w + \frac{1 - 4i}{2}\right)\left(w + \frac{1 + 4i}{2}\right)$ (h) $3(w - 1 + i)(w - 1 - i)$ (i) $-2\left(w - 2 - \frac{\sqrt{6}i}{2}\right)\left(w - 2 + \frac{\sqrt{6}i}{2}\right)$

2. (a) $-2 \pm 2i$ (b) $\frac{1 \pm \sqrt{11}i}{2}$ (c) $\frac{3 \pm \sqrt{3}i}{6}$ (d) $\frac{-5 \pm \sqrt{7}i}{4}$ (e) $-5 \pm 2i$ (f) $\pm 4i$ (g) $-6 \pm 2i$
 (h) $-3 \pm i$ (i) $\pm \frac{5}{3}i$ **3.** (a) $\pm 2, \pm i$ (b) $\pm 3, \pm i$ (c) $\pm 3, \pm 2i$ **4.** (a) $(z - 5i)(z + 5i)$

(b) $(z - 7i)(z + 7i)$ (c) $(z + 2 + i)(z + 2 - i)$ (d) $(z + 3 + \sqrt{2}i)(z + 3 - \sqrt{2}i)$
 (e) $(z - 2i)(z + 2i)(z - \sqrt{2})(z + \sqrt{2})$ (f) $(z - \sqrt{2}i)(z + \sqrt{2}i)(z - \sqrt{3})(z + \sqrt{3})$

EXERCISE 11.4.2

1. (a) $(z + 2)(z + i)(z - i)$ (b) $(z - 9)(z + i)(z - i)$ (c) $(z - 2)(z + \sqrt{2}i)(z - \sqrt{2}i)$
2. (a) $(w + 1 - \sqrt{5}i)(w + 1 + \sqrt{5}i)(w - 2)$ (b) $(z - 1)(z - 2 + i)(z - 2 - i)$
 (c) $(z - 1)(z + 1 + i)(z + 1 - i)$ (d) $(x + 2)(x - 2)(x + i)(x - i)$
 (e) $(w + 2)(w - 1 + i)(w - 1 - i)$ (f) $(z + 5)(z - 5)(z + 5i)(z - 5i)$ **3.** (a) $1, 3 \pm 4i$ (b) $2, 3 \pm 2i$
 (c) $-2, 3, 1 \pm i$ (d) $\frac{1}{2}, -1 \pm i$ (e) $-\frac{5}{3}, \frac{3}{2}, 1 \pm \sqrt{2}i$ (f) $-1, -3 \pm i$ **4.** $\frac{1}{3}, \frac{-1 \pm \sqrt{3}i}{2}$ **5.** $-\frac{1}{2}, 1 \pm 2i$
6. $(z - 3)(z - 2 + 3i)(z - 2 - 3i)$ **7.** $1 \pm 2i, \frac{-1 \pm \sqrt{11}i}{2}$ **8.** (a) $(2z - 1)(z + i)(z - i)$
 (b) $(z + \sqrt{3})(z - \sqrt{3})(z + 2i)(z - 2i)$ **9.** $2 \pm i, -1$ **10.** $2 \pm 3i, -\frac{13}{4}$ **11.** $2 \pm i, \frac{1}{2}$
12. $(z - 2)(z - 4 + i)(z - 4 - i)$ **13.** $(z - 2)(z - 1 + i)(z - 1 - i)$
14. (a) $-2, 1, \frac{-1 \pm \sqrt{3}i}{2}, 1 \pm \sqrt{3}i$ (b) $1, 2, \frac{1 \pm \sqrt{3}i}{2}, -1 \pm \sqrt{3}i$ (c) $\pm \sqrt{3}, \pm i$ (d) $\pm \sqrt{5}, \pm i$
15. (a) $x^3 - 7x^2 + 17x - 15 = 0$ (b) $x^4 - 5x^3 + 10x^2 - 10x + 4 = 0$
 (c) $x^3 - 5x^2 + 10x - 12 = 0$ (d) $x^4 + 2x^3 + 2x^2 - 2x + 21 = 0$ **16.** $-1 - i\sqrt{3}, 1 \pm \sqrt{5}$
17. $2 \pm i, \frac{1}{2}(-3 \pm \sqrt{5})$

EXERCISE 11.4.3

1. (a) $-\frac{3}{2} \pm \frac{3\sqrt{3}}{2}i, 3$ (b) $\pm \frac{3\sqrt{3}}{2} + \frac{3}{2}i, -3i$ (c) $2i, \pm \sqrt{3} - i$
 (d) $-\sqrt{2} - \sqrt{2}i, -\sqrt{2} + \sqrt{2}i, \sqrt{2} - \sqrt{2}i, \sqrt{2} + \sqrt{2}i$
 (e) $\frac{3}{2}(\sqrt{2} - \sqrt{2} - \sqrt{2} + \sqrt{2}i), -\frac{3}{2}(\sqrt{2} - \sqrt{2} - \sqrt{2} + \sqrt{2}i), \frac{3}{2}(\sqrt{2} + \sqrt{2} + \sqrt{2} - \sqrt{2}i)$
 $-\frac{3}{2}(\sqrt{2} + \sqrt{2} + \sqrt{2} - \sqrt{2}i)$ (f) $\pm 2, \sqrt{3} \pm i, -\sqrt{3} \pm i$ **2.** $1 \pm i, -1 \pm i;$
 $(z - 1 - i)(z - 1 + i)(z + 1 - i)(z + 1 + i)$ **3.** (a) $\pm \frac{1}{\sqrt{2}}(1 + i)$ (b) $2 + i, -2 - i$ (c) $\pm \frac{\sqrt{2}}{2}(1 + \sqrt{3}i)$
4. (a) $\sqrt[6]{2}cis\left(-\frac{\pi}{12}\right), \sqrt[6]{2}cis\left(\frac{7\pi}{12}\right), \sqrt[6]{2}cis\left(-\frac{9\pi}{12}\right)$ (b) $\sqrt[3]{2}cis\left(\frac{2\pi}{9}\right), \sqrt[3]{2}cis\left(\frac{8\pi}{9}\right), \sqrt[3]{2}cis\left(-\frac{4\pi}{9}\right)$
 (c) $cis\left(\frac{\pi}{6}\right), cis\left(\frac{5\pi}{6}\right), cis\left(-\frac{\pi}{2}\right)$ **5.** (a) $\sqrt[8]{2}cis\left(\frac{\pi}{16}\right), \sqrt[8]{2}cis\left(\frac{9\pi}{16}\right), \sqrt[8]{2}cis\left(-\frac{15\pi}{16}\right), \sqrt[8]{2}cis\left(-\frac{7\pi}{16}\right)$
 (b) $cis\left(\frac{\pi}{8}\right), cis\left(\frac{5\pi}{8}\right), cis\left(\frac{9\pi}{8}\right), cis\left(\frac{13\pi}{8}\right)$ (c) $i, \pm \frac{\sqrt{3}}{2} - \frac{1}{2}i$

(d) $2cis\left(-\frac{\pi}{12}\right), 2cis\left(\frac{5\pi}{12}\right), 2cis\left(\frac{11\pi}{12}\right), 2cis\left(-\frac{7\pi}{12}\right)$ (e) $2(\pm\sqrt{3} + 1i), -4i$

(f) $\pm\frac{1}{2}((\sqrt{3} + 1) + (\sqrt{3} - 1)i)$ **6.** (a) $1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ **7.** $-2, 1 \pm \sqrt{3}i$

EXERCISE 12.4.1

1. (a) $2n + 2$ (b) $5n - 2$ (c) $\frac{1}{n}$ (d) 3^n (e) $n^2 + 1$ (f) $\frac{n}{n+1}$ (g) $\frac{n+1}{n}$ (h) 2^{3-n} **2.** (i) $n^2 + 2n + 2$
 (ii) $2n^2 + n - 2$ (iii) $n^3 - 2n$ (iv) $\frac{20 + n^2 - n^4}{4}$ (v) $1 + n + 6n^2 - n^3$

EXERCISE 12.4.2

1. (a) $\frac{n(2n^2 + 3n + 7)}{6}$ (b) $\frac{n^2(n+1)^2}{4}$ (c) $\frac{1}{4}\left(1 - \frac{1}{5n}\right)$ (d) $n^2(2n^2 - 1)$ (e) $\frac{n(2n^2 + 9n + 7)}{6}$
 (f) $\frac{n}{2n+1}$ **2.** $\frac{n(n+1)}{2}$ **3.** $\frac{180(n-2)}{n}$ **4.** $\frac{n^2 + n + 2}{2}$ **5.** $\frac{n(n+1)(2n+1)}{6}$ **7.** $2n^2 - 2n + 1$
8. $\frac{n^4 - 6n^3 + 23n^2 - 18n + 24}{24}$ **9.** $15, \frac{n(n+1)}{2}, \frac{n(n+2)}{2}, \frac{3n(n+1)}{2}$ **12.** $2n + 1$

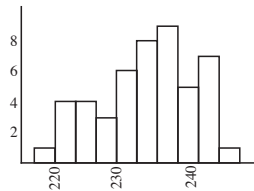
EXERCISE 13.1

1. (a) i. 14 500 ii. 2 000 (b) 305 (304.5) **2.** Sample size is large but may be biased by factors such as the location of the catch. Population estimate is 5 000. **3.** (a) i. 1500 ii. 120 (b) 100 (c) 1 000 **4.** (a), (c) numerical, (b), (d), (e) categorical **5.** (a), (d) discrete, (b), (c), (e) continuous

EXERCISE 13.2

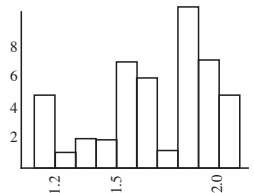
1.

218-220	221-223	224-226	227-229	230-232	233-235	236-238	239-241	242-244	245-247
1	4	4	3	6	8	9	5	7	1



2.

1.1-1.2	1.2-1.3	1.3-1.4	1.4-1.5	1.5-1.6	1.6-1.7	1.7-1.8	1.8-1.9	1.9-2.0	2.0-2.1
5	1	2	2	7	6	1	12	7	5



3. Set A Mode = 29.1 Mean = 27.2 Median = 27.85
 Set B Mode = 9 Mean = 26.6 Median = 9. Set B is much more spread out than set A and although the two sets have a similar mean, they have very different mode and median.

EXERCISE 13.3

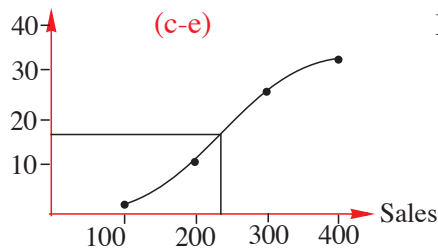
1. Mode = 236-238gms; Mean = 234gms; Median = 235gms **2.** Mode = 1.8-1.9gms; Mean = 1.69gms Median = 1.80gms **3.** Set A Mode = 29.1 Mean = 27.2 Median = 27.85 Set B Mode = 9 Mean = 26.6 Median = 9. **4.** (a) \$27522 (b) \$21025 (c) Median **5.** i. \$233 300 ii. \$169 000 iii. Median **6.** (a) 14.375 (b) 14.354

EXERCISE 13.4

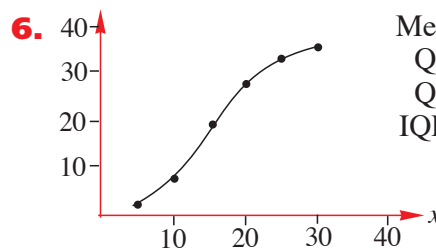
1. (a) Sample A Mean = 1.99kg Sample B Mean = 2.00kg (b) Sample A Sample std = 0.0552kg Sample B Sample std = 0.1877kg (c) Sample A Population std = 0.0547kg Sample B Population std = 0.1858kg 2. (a) 16.4 (b) 6.83 3. Mean = 49.97 Std = 1.365

EXERCISE 13.5

1. (a) Med = 5, Q1 = 2, Q3 = 7, IQR = 5 (b) Med = 3.3, Q1 = 2.8, Q3 = 5.1, IQR = 2.3 (c) Med = 163.5, Q1 = 143, Q3 = 182, IQR = 39 (d) Med = 1.055, Q1 = 0.46, Q3 = 1.67, IQR = 1.21 (e) Med = 5143.5, Q1 = 2046, Q3 = 6252, IQR = 4206
 2. (a) Med = 3, Q1 = 2, Q3 = 4, IQR = 2 (b) Med = 13, Q1 = 12, Q3 = 13, IQR = 1 (c) Med = 2, Q1 = 2, Q3 = 2.5, IQR = 0.5 (d) Med = 40, Q1 = 30, Q3 = 50, IQR = 20 (e) Med = 20, Q1 = 15, Q3 = 22.5, IQR = 7.5
 3. (a) \$84.67 (b) \$148 (c) \$11 (d) Q1 = \$4.50, Q3 = \$65 IQR = \$60.50 (e) Median & IQR.
 4. (a) 2.35 (b) 1.25 (c) 2 (d) Q1 = 1, Q3 = 3, IQR = 2 5. (a) \$232 (b) \$83



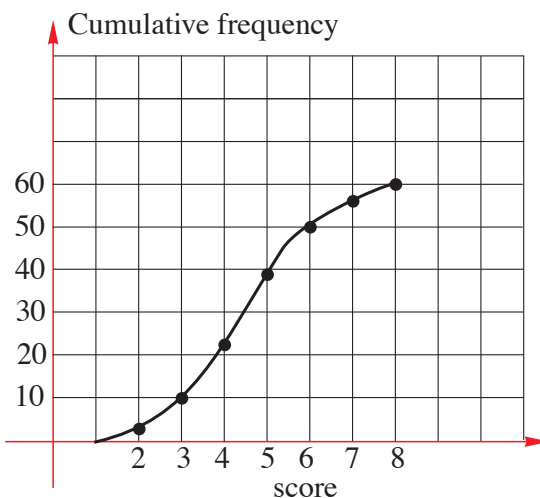
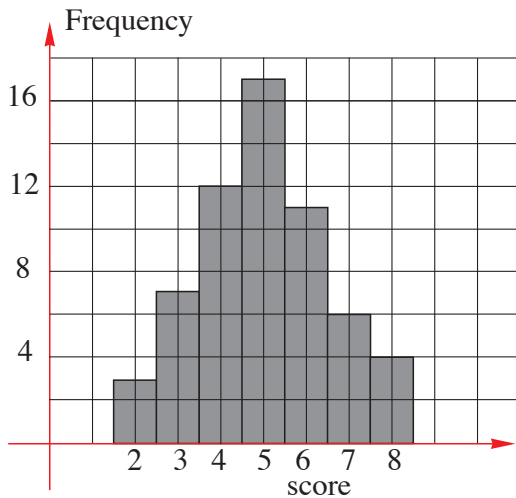
Med = \$220
 Q1 = \$160
 Q3 = \$310
 IQR = \$150



Med = 16
 Q1 = 10
 Q3 = 24
 IQR = 14

EXERCISE 13.6

1. (a) Sample–100 randomly selected patients, population – all suffering from AIDS
 (b) Sample–1000 working aged people in N.S.W, population – all working aged people in N.S.W
 (c) Sample – John’s I.B Higher Maths class, population – all seniors at Nappa Valley High School.
 2. Discrete: a, b, d; Continuous: c, e, f, g.
 3. (b)



4. suggested answers only: (a) 200–224; 225–249; 250–274; ... 575–599
 (b) 100–119; 120–139; ... 400–419 (c) 440–459; 460–479; ... 780–799.
 5. Make use of your graphics calculator
 6. (a) 16 (b) graphics calculator (c) 15.23
 (d) 15.5 (e) Q1 = 14, Q3 = 17 (f) 15.87 (2dec.pl.)
 7. (a) 30–34 (b) graphics calculator (c) 30.4
 (d) 32 (approx)
 8. (b) 215.5 (c) 216.2 9. 48.17

- 10.** (a) $Q1 \sim 35, Q3 \sim 95$ (b) ~ 105 (c) 61% (d) 67.15
11. range = 19, $s = 5.49$ **12.** 5.8; 1.50 **13.** 17.4; $s_n = 3.12$ $s_{n-1} = 3.18$
14. (a) 6 (b) 7 (c) $Q1 = 5, Q3 = 7$
 (d) 2 (e) 6.15 (f) 1.61
15. $s_n = 18.8, s_{n-1} = 19.1$ **16.** 14.18

EXERCISE 14.1

- 1.** 15 **2.** (a) 25 (b) 625 **3.** (a) 24 (b) 256 **4.** (a) 24 (b) 48 **5.** 15 **6.** 270 **7.** 120 **8.** 336 **9.** 60
10. (a) 362880 (b) 80640 (c) 1728 **11.** 20 **12.** (a) $10!$ (b) $2 \times 8!$ (c) i. $2 \times 9!$ ii. $8 \times 9!$ **13.** 34650
14. 4200 **15.** 4

EXERCISE 14.2

- 1.** 792 **2.** (a) 1140 (b) 171 **3.** 1050 **4.** 70 **5.** 2688 **6.** (a) 210 (b) 420 **7.** 24000 **8.** 8 **9.** 155
10. 5

EXERCISE 14.3

- 1.** (a) 120 (b) 325 **2.** 5040 **3.** (a) 144 (b) 1440 **4.** (a) 720 (b) 240 **5.** 11760 **6.** 7056; 4606
7. (a) 840 (b) 1680 **8.** 190 **9.** 10080 **10.** 226800 **11.** (a) 715 (b) 315 (c) 665 **13.** ${}^n C_2$ **14.** ${}^n C_4$
15. (b) 92 **16.** 252 **17.** (a) 1287 (b) 560 **18.** 256 **19.** 288 **20.** (a) 10080 (b) 30240 (c) 14400
21. 10080, 1080 **22.** 3528000 **23.** 720; 240 **24.** 103680 **25.** (a) 12 (b) 128 **26.** 2880
27. (a) 30030 (b) 37310 **28.** 77055 **29.** (a) 48 (b) 72

EXERCISE 15.1

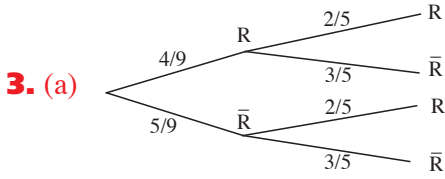
- 1.** (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ **2.** (a) $\frac{2}{7}$ (b) $\frac{5}{7}$ **3.** (a) $\frac{5}{26}$ (b) $\frac{21}{26}$ **4.** {HH, HT, TH, TT} (a) $\frac{1}{4}$ (b) $\frac{3}{4}$
5. {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT} (a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ **6.** (a) $\frac{2}{9}$ (b) $\frac{2}{9}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$
7. (a) $\frac{1}{2}$ (b) $\frac{3}{10}$ (c) $\frac{9}{20}$ **8.** (a) $\frac{11}{36}$ (b) $\frac{1}{18}$ (c) $\frac{1}{6}$ (d) $\frac{5}{36}$ **9.** {GGG, GGB, GBG, BGG, BBB, BBG, BGB, GBB} (a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ **10.** (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4}$ **11.** (a) $\frac{3}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$
12. (a) {(1, H), (2, H), (3, H), (4, H), (5, H), (6, H), (1, T), (2, T), (3, T), (4, T), (5, T), (6, T)}
 (b) $\frac{1}{4}$ **13.** (a) $\frac{1}{216}$ (b) $\frac{1}{8}$ (c) $\frac{3}{8}$

EXERCISE 15.2

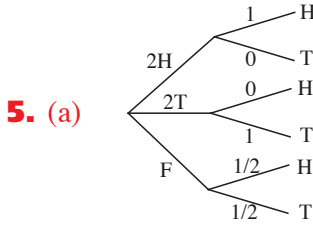
- 1.** (a) $\frac{1}{4}$ (b) $\frac{5}{8}$ (c) $\frac{3}{4}$ **2.** (a) $\frac{1}{13}$ (b) $\frac{1}{2}$ (c) $\frac{1}{26}$ (d) $\frac{7}{13}$ **3.** $\frac{9}{26}$ **4.** (a) 1.0 (b) 0.3 (c) 0.5 **5.** (a) 0.65
 (b) 0.70 (c) 0.65 **6.** (a) 0.95 (b) 0.05 (c) 0.80 **7.** (a) {TTT, TTH, THT, HTT, HHH, HHT, HTH, THH} (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$ (e) $\frac{3}{8}$ **8.** (a) $\frac{6}{25}$ (b) $\frac{6}{25}$ (c) $\frac{13}{25}$ **9.** (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$ (e) $\frac{7}{12}$
10. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{8}{13}$ (d) $\frac{7}{13}$ **11.** (a) 0.1399 (b) 0.8797 (c) 0.6 **12.** (b) $\frac{4}{15}$ (c) $\frac{4}{15}$ (d) $\frac{11}{15}$

EXERCISE 15.3

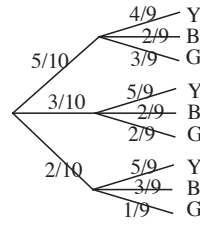
1. (a) 0.7 (b) 0.75 (c) 0.50 (d) 0.5 2. (a) 0.5 (b) 0.83 (c) 0.10 (d) 0.90



(b) $\frac{8}{45}$ (c) $\frac{22}{45}$ (d) $\frac{6}{11}$ 4. (a) 0.5 (b) 0.30 (c) 0.25



(b) $\frac{1}{2}$ (c) $\frac{2}{3}$ 6. $\frac{1}{3}$ 7. (a)



(b) $\frac{31}{45}$ (c) $\frac{2}{9}$ 8. $\frac{2}{3}$

9. (a) 0.88 (b) 0.42 (c) 0.6 (d) 0.28 10. (a) 0.33 (b) 0.49 (c) 0.82 (d) 0.551 11. (a) 0.22 (b) 0.985 (c) 0.8629 12. (a) 0.44 (b) 0.733 14. (a) 0.512 (b) 0.128 (c) 0.8571 15. (a) 0.2625 (b) 0.75 (c) 0.4875 (d) 0.7123 16. (a) 0.027 (b) 0.441 (c) 0.453

EXERCISE 15.4

1. (a) 0.042 (b) 0.7143 2. (a) 0.4667 (b) 0.3868 3. (a) $\frac{5}{7}$ (b) $\frac{9}{13}$ 4. $\frac{5}{9}$ 5. (b) i. $\frac{1}{40}$ ii. 0.2

6. (a) i. $\frac{2N-m}{2N}$ ii. $\frac{2(N-m)}{2N-m}$ (b) $\frac{m}{m+(N-m)2^n}$ 7. $\frac{9}{19}$ 8. a. 0.07 b. 0.3429 c. 0.30

d. 0.0282 9. a. 0.8008 b. 0.9767 c. 0.0003 10. a. 0.0464 b. 0.5819 c. 0.9969

11. a. 0.2 b. 0.08 c. 0.72 d. $\begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$; $\begin{bmatrix} 0.72 & 0.28 \\ 0.56 & 0.44 \end{bmatrix}$ g. 0.688

EXERCISE 15.5

1. (a) $\frac{5}{126}$ (b) $\frac{5}{18}$ (c) $\frac{1}{126}$ 2. (a) $\frac{1}{5}$ (b) $\frac{1}{10}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$ 3. (a) $\frac{72}{5525}$ (b) $\frac{1}{5525}$ (c) $\frac{1}{1201}$ 4. $\frac{2}{5}$

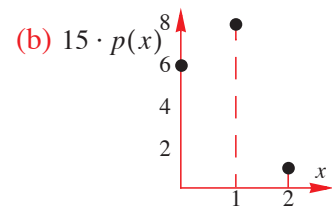
5. (a) $\frac{63}{143}$ (b) $\frac{133}{143}$ 6. (a) $\frac{5}{12}$ (b) $\frac{5}{33}$ (c) $\frac{5}{6}$ 7. $\frac{3}{11}$ 8. (a) $\frac{4}{13}$ (b) $\frac{9}{13}$ 9. (a) $\frac{67}{91}$ (b) $\frac{22}{91}$

10. (a) $\frac{1}{4}$ (b) $\frac{1}{28}$ (c) $\frac{5}{14}$ 11. (a) $\frac{5}{28}$ (b) $\frac{1}{28}$ 12. $\frac{6}{13}$ 13. (a) $\frac{1}{6}$ (b) $\frac{1}{4}$ 14. (a) $\frac{1}{210}$ (b) $\frac{7}{9}$

15. (a) $\frac{7}{1938}$ (b) 0.6 16. $\frac{11}{21}$

EXERCISE 16.1

1. 0.3 2. (a) 0.1 (b) i. 0.2 ii. 0.7 3. (a) $p(0) = \frac{6}{15}$, $p(1) = \frac{8}{15}$, $p(2) = \frac{1}{15}$ (c) $\frac{14}{15}$

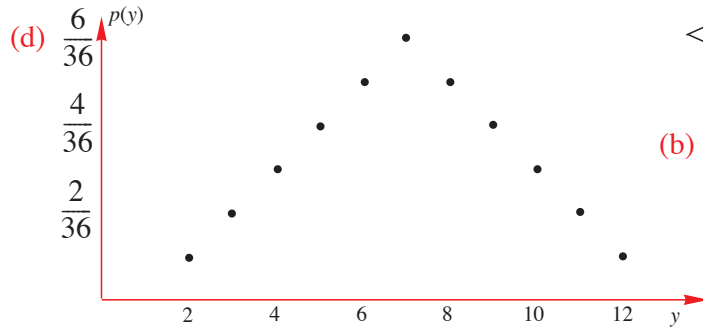


4. (a) {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

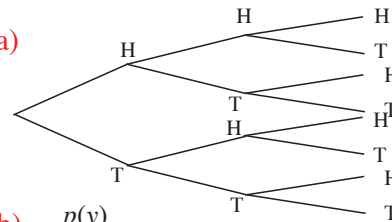
(b)

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

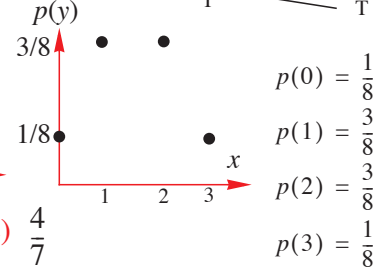
(c) $\frac{5}{36}$



5. (a)

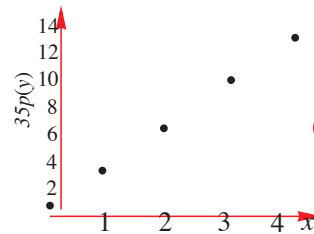


(b)



(c) $\frac{4}{7}$

6. (a) $\frac{1}{35}$ (b) i. $p(2) = \frac{7}{35}$ $p(3) = \frac{10}{35}$ ii. $p(4) = \frac{13}{35}$



(c) $\frac{6}{7}$ 7. (a) i. 0.9048

ii. 0.09048 (b) 0.0002 8. 0.3712 9. $p(0) = \frac{11}{30}$, $p(-1) = \frac{1}{2}$, $p(3) = \frac{2}{15}$ (b) i. $\frac{11}{30}$ ii. $\frac{13}{15}$

10.

n	0	1	2
$P(N=n)$	$\frac{6}{15}$	$\frac{8}{15}$	$\frac{1}{15}$

11. (a)

n	1	2	3	4
$P(N=n)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(b)

s	2	3	4	5	6	7	8
$P(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

12. (a) 0.81 (b) 0.2439

EXERCISE 16.2

1. (a) 2.8 (b) 1.86 2. (a) 3 (b) i. 1 ii. 1 (c) i. 6 ii. 0.4 3. (a) i. 1.3 ii. 2.5 iii. -0.1 (b) i. 0.9

ii. 7.29 (c) i. $\frac{31}{60}$ ii. 0.3222 4. $\mu = \frac{2}{3}$, $\sigma^2 = 0.3556$ 5. (a) 7 (b) 5.8333 6. $np = 3 \times \frac{1}{2} = 1.5$

7. (a) $\frac{1}{25}$ (b) 2.8 (c) 1.166 8. (a) 0.1 (b) i. 0.3 ii. 1 (c) i. 0 ii. 1 iii. 2 9. 5.56

10. $p(0) = \frac{35}{120}$, $p(1) = \frac{63}{120}$, $p(2) = \frac{21}{120}$, $p(3) = \frac{1}{120}$ (b) i. 0.9 ii. 0.49 (c) $W = 3N - 3$

$E(W) = -0.3$ 11. (a) \$ -1.00 (b) both the same 12. (a) 50 (b) 18 (c) 2 13. (a) 11 (b) $\frac{\sqrt{3}}{3}$ (c) -4

14. (a) 0.75 (b) 0.6339 15. (a) $E(X) = 1 - 2p$, $\text{Var}(X) = 4p(1 - p)$ (b) i. $n(1 - 2p)$ ii. $4np(1 - p)$

16.

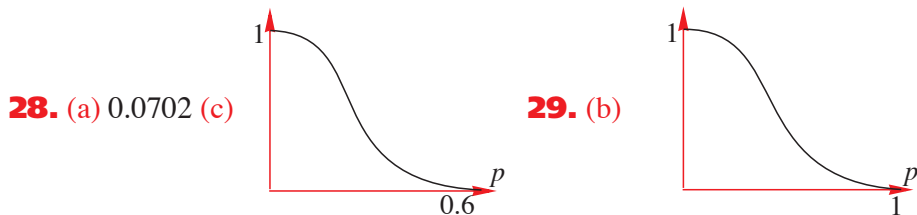
n	0	1	2
$P(N=n)$	$\frac{28}{45}$	$\frac{16}{45}$	$\frac{1}{45}$

$W = 21.43$ 17. (a) $a = \frac{2}{3}$, $0 \leq b \leq 1$

(b) $E(X) = \frac{b+1}{3}$, $Var(X) = \frac{1}{9}(2+7b-b^2)$ **18.** (a) $E(X) = 4$, $Var(X) = 20$

EXERCISE 16.3

- 1.** (a) 0.2322 (b) 0.1737 (c) 0.5941 **2.** (a) 0.3292 (b) 0.8683 (c) 0.2099 (d) 0.1317 **3.** (a) 0.1526 (b) 0.4812 (c) 0.5678 **4.** (a) 0.7738 (b) 3.125×10^{-7} (c) 0.9988 (d) 3×10^{-5} **5.** (a) 0.2787 (b) 0.4059 **6.** (a) 0.2610 (b) 0.9923 **7.** (a) 0.2786 (b) 0.7064 (c) 0.1061 **8.** (a) 0.1318 (b) 0.8484 (c) 0.0549 (d) 0.3296 **9.** (a) 0.2938 (b) 0.6531 (c) 0.0027 (d) 0.9726 (e) 12.86 **10.** (a) 0.0039 (b) 0.2734 (c) 0.6367 (d) 0.9648 **11.** (a) 0.3125 (b) 0.0156 (c) 0.3438 (d) 3 **12.** (a) 0.2785 (b) 0.3417 (c) 120 **13.** (a) 0.0331 (b) 0.5695 **14.** (a) 0.4305 (b) 0.9619 (c) \$720 (d) 0.2059 **15.** (a) i. 1.4 ii. 1 iii. 1.058 iv. 0.0795 v. 0.0047 (b) i. 3.04 ii. 3 iii. 1.373 iv. 0.2670 v. 0.1390 **16.** 38.23 **19.** (a) i. 0.1074 ii. 7.9×10^{-4} iii. 0.3758 (b) at least 6 **20.** (a) $\frac{4}{3}$ (b) $\frac{10}{9}$ (c) $\frac{1}{6}$ (d) $\frac{5}{288}$ **21.** (a) 20 (b) 3.4641 **22.** (a) 102.6 (b) 0.000254 **23.** (a) i. 6 ii. 2.4 (b) i. 6 ii. 3.6 **24.** 0.1797 **25.** 1.6, 1.472 **26.** (a) 0.1841 (b) \$11.93 **27.** (a) \$8 (b) \$160



30. (b) 0.8035 (c) 39.3 **31.** (a) $\frac{np - np(1-p)^{n-1}}{1 - (1-p)^n - np(1-p)^{n-1}}$, $0 < p < 1$

EXERCISE 16.4

- 1.** (a) 0.3263 (b) 0.0932 **2.** (a) 0.0015 (b) 0 (c) 0.9714 **3.** (a) 1.2 (b) 0.56 **4.** (a) 3.4848 (b) $\frac{10}{3}$ (c) 0.7071 **5.** (a) $\frac{1}{1365}$ (b) $\frac{22}{91}$ **6.** $\frac{1921}{1938}$ **7.** (a) $\frac{27}{91}$ (b) $\frac{87}{91}$

8. (a)

x	1	2	3	4	5
$P(X=x)$	0.0238	0.2381	0.4762	0.2381	0.0238

 (b) 3 **9.** (a) $\frac{9}{22}$ (b) $\frac{1}{22}$ **10.** (a) $\frac{46}{255}$

- (b) $\frac{184}{595}$ (c) $\frac{32}{357}$ **11.** (a) $\frac{7}{22}$ (b) $\frac{37}{44}$ **12.** $\frac{49}{60}$ **13.** (a) hypergeometric

(b)

x	0	1	2	3	4
$P(X=x)$	$\frac{330}{1365}$	$\frac{660}{1365}$	$\frac{330}{1365}$	$\frac{44}{1365}$	$\frac{1}{1365}$

 (c) $\frac{66}{91}$ **14.** $\frac{1}{14}$ **15.** (a) $\frac{1}{8}$ (b) $\frac{47}{72}$

- 16.** (a) $\frac{44}{45}$ (b) 0.9999 **17.** $\frac{11}{21}$ **18.** 0.8 **19.** (a) $\frac{48}{55}$ (b) i. 0.3365 ii. 0.01066 (c) 0.9682

- 20.** (a) $\frac{28}{45}$ (b) $\frac{16}{45}$ (c) $\frac{1}{45}$, 10 days beforehand (place order on 11 July)

22. (a)

x	1	2	3	4	5
$P(X=x)$	0.598	0.315	0.075	0.010	0.001

 remainder ~ 0

(b)

x	0	1	2	3	4	5	6	7	8	9	10
$P(\text{Accept})$	1	$\frac{4}{5}$	$\frac{28}{45}$	$\frac{7}{15}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{1}{45}$	0	0

, $\sim 90\%$

EXERCISE 16.5

1. (a) $P(X = x) = \frac{e^{-2}2^x}{x!}, x = 0, 1, 2, \dots$ (b) i. 0.1353 ii. 0.2707 iii. 0.5940 iv. 0.4557
 2. (a) 0.0383 (b) 0.1954 3. (a) 0.2052 (b) 0.9179 4. (a) 0.2623 (b) 0.8454 5. (a) 0.0265
 (b) 0.0007 6. (a) 0.1889 (b) 0.7127 7. (a) 0.7981 (b) 0.2019 (c) 0.1835 8. (a) 0.2661
 (b) 0.5221 9. 0.1912 10. (a) 0.3504 (b) 0.6817 11. (a) 0.00127 (b) 0.0500 12. (a) 0.1804
 (b) 0.0166 (c) 0.3233 13. (a) 0.8131; 0.5511 No 14. 0.4781 15. (a) 0.3679 (b) 0.2642
 (c) 0.2135 16. (a) i. p ii. $-p \ln p$ iii. $1 - p + p \ln p$ (c) 0.4785

EXERCISE 17.1

1. (i) 0.6915 (ii) 0.9671 (iii) 0.9474 (iv) 0.9965 (v) 0.9756 (vi) 0.0054 (vii) 0.0287
 (viii) 0.0594 (ix) 0.0073 (x) 0.8289 (xi) 0.6443 (xii) 0.0823 2. (i) 0.0360 (ii) 0.3759
 (iii) 0.0623 (iv) 0.0564 (v) 0.0111 (vi) 0.2902 (vii) 0.7614 (viii) 0.0343 (ix) 0.6014 (x) 0.1450
 (xi) 0.9206 (xii) 0.2668 (xiii) 0.7020 (xiv) 0.9132 (xv) 0.5203 (xvi) 0.8160 (xvii) 0.9388
 (xviii) 0.7258

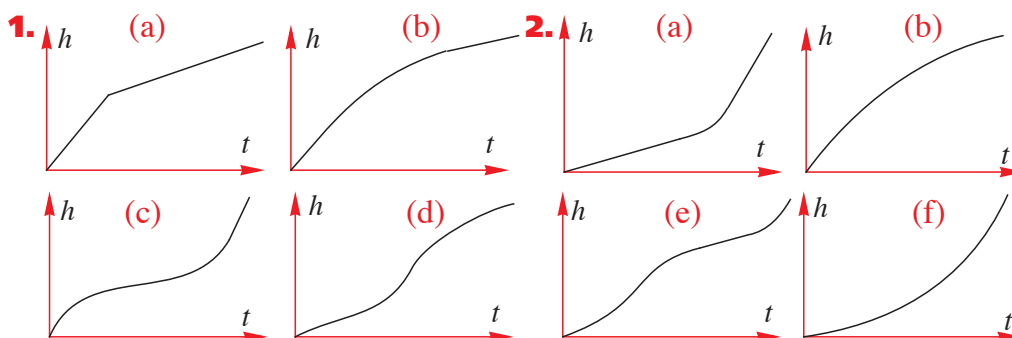
EXERCISE 17.2

1. (a) 0.0228 (b) 0.9332 (c) 0.3085 (d) 0.8849 (e) 0.0668 (f) 0.9772 2. (a) 0.9772 (b) 0.0668
 (c) 0.6915 (d) 0.1151 (e) 0.9332 (f) 0.0228 3. (a) 0.3413 (b) 0.1359 (c) 0.0489 4. (a) 0.6827
 (b) 0.1359 (c) 0.3934 5. (a) 0.8413 (b) 0.4332 (c) 0.7734 6. (a) 0.1151 (b) 0.1039 (c) 0.1587
 7. (a) 0.1587 (b) 0.6827 (c) 0.1359 8. (a) 0.1908 (b) 0.4754 (c) 16.88 9. (a) 0.1434 (b) 0.6595
 10. (a) 0.2425 (b) 0.8413 (c) 0.5050 11. (a) -1.2816 (b) 0.2533 12. (a) 58.2243 (b) 41.7757
 (c) 59.80 13. 39.11 14. 9.1660 15. 42% 16. 0.7021 17. (i) 0.2903 (ii) 0.4583 (iii) 0.2514
 18. 23% 19. 0.5 20. 11% 21. 5% 22. 14% 23. 1.8 24. 252 25. 0.1517 26. 0.3821
 27. 0.22 28. 322 29. 0.1545 30. 7 31. 87 32. (a) i. 0.0062 ii. 0.0478 iii. 0.9460 (b) 0.0585
 33. (a) \$5.11 (b) \$7.39 34. (a) 0.0062 (b) i. 0.7887 ii. 0.0324 (c) \$1472
 35. (a) $\mu = 66.86, \sigma = 10.25$ (b) \$0.38\$ 36. (a) $\mu = 37.2, \sigma = 28.2$ (b) 20 (19.9)
 37. (a) i. 0.3446 ii. 0.2347 (b) i. 0.3339 ii. 0.3852 (c) 0.9995

EXERCISE 18.1

1. (a) $\frac{3}{4}$ (b) $\frac{3a}{4b}$ (c) -1 (d) 1 (e) $-\frac{15}{8}$ (f) 0 2. (a) 4 (b) 0.2 (c) 0.027 (d) 0.433 (e) -0.01 (f) 6.34
 (g) 6.2 (h) 0 3. (a) 6 m/s (b) 30 m/s (c) $11 + 6h + h^2$ m/s 4. 12 m/s 5. $8 + 2h$ 6. -3.49 °C/sec
 7. (a) 127π cm³/cm (b) i. 19.6667π cm³/cm ii. 1.9967π cm³/cm iii. 0.2000π cm³/cm 8. 1.115
 9. (a) -7.5 °C/min (b) $t = 2$ to $t = 6$ 10. (a) 28 m (b) 14 m/s (c) average speed (d) 49 m
 (e) 49 m/s 11. (a) \$1160, \$1345.6, \$1560.90, \$1810.64, \$2100.34 (b) \$220.07 per year

EXERCISE 18.2



EXERCISE 18.3

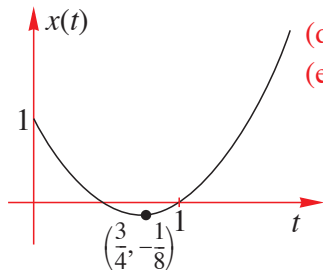
1. (a) $h + 2$ (b) $4 + h$ (c) $\frac{-1}{1+h}$ (d) $3 - 3h + h^2$ 2. (a) 2 (b) 4 (c) -1 (d) 3 3. (a) $2a + h$

(b) $-(2a + h)$ (c) $(2a + 2) + h$ (d) $3a^2 + 1 + 3ah + h^2$ (e) $-(3a^2 + 3ah + h^2)$

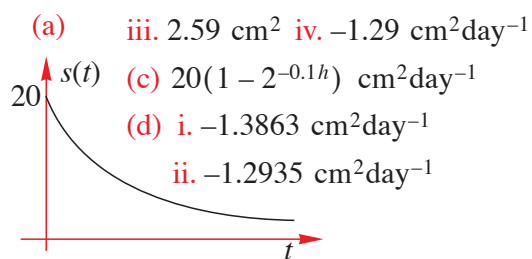
(f) $3a^2 - 2a + (3a - 1)h + h^2$ (g) $\frac{-2}{a(a+h)}$ (h) $-\frac{1}{(a-1)(a-1+h)}$ (i) $\frac{1}{\sqrt{a+h} + \sqrt{a}}$

4. (a) 1 ; 1 (b) $2a + h$; $2a$ (c) $3a^2 + 3ah + h^2$; $3a^2$ (d) $4a^3 + 6a^2h + 4ah^2 + h^3$; $4a^3$

5. (a) (b) i. 3 ms^{-1} ii. 2 ms^{-1} iii. 1.2 ms^{-1} 6. (b) i. 20 cm^2 ii. 17.41 cm^2



(d) Find (limit) as $h \rightarrow 0$
(e) $4t - 3$



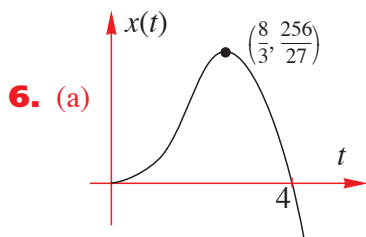
(a) iii. 2.59 cm^2 iv. $-1.29 \text{ cm}^2 \text{ day}^{-1}$
(c) $20(1 - 2^{-0.1h}) \text{ cm}^2 \text{ day}^{-1}$
(d) i. $-1.3863 \text{ cm}^2 \text{ day}^{-1}$
ii. $-1.2935 \text{ cm}^2 \text{ day}^{-1}$

EXERCISE 18.4

1. (a) 3 (b) 8 (c) $-\frac{1}{9}$ (d) 1.39 (e) -1 (f) $\frac{17}{16}$ 2. 4.9 m (b) $4.9(h^2 + 2h)$ m (c) 9.8 m/s

3. (a) $8x$ (b) $10x$ (c) $12x^2$ (d) $15x^2$ (e) $16x^3$ (f) $20x^3$ 4. (a) $4x$ (b) -1 (c) $-1 + 3x^2$ (d) $-x^{-2}$

(e) $-2(x+1)^{-2}$ (f) $0.5x^{-1/2}$ 5. (a) 1 ms^{-1} (b) $(2-a) \text{ ms}^{-1}$



6. (a) (b) i. 5 ms^{-1} ii. 4 ms^{-1} (c) $8t - 3t^2 \text{ ms}^{-1}$ (d) $\frac{8}{3} \text{ sec}$

EXERCISE 19.1

1. (a) $5x^4$ (b) $9x^8$ (c) $25x^{24}$ (d) $27x^2$ (e) $-28x^6$ (f) $2x^7$ (g) $2x$ (h) $20x^3 + 2$ (i) $-15x^4 + 18x^2 - 1$

(j) $-\frac{4}{3}x^3 + 10$ (k) $9x^2 - 12x$ (l) $3 + \frac{2}{5}x + 4x^3$ 2. (a) $-\frac{3}{x^4}$ (b) $\frac{3}{2}\sqrt{x}$ (c) $\frac{5}{2}\sqrt{x^3}$ (d) $\frac{1}{3^3\sqrt{x^2}}$ (e) $\frac{2}{\sqrt{x}}$

(f) $9\sqrt{x}$ (g) $\frac{1}{\sqrt{x}} + \frac{3}{x^2}$ (h) $\frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x^3}}$ (i) $\frac{10}{3^3\sqrt{x}} - 9$ (j) $5 - \frac{1}{2\sqrt{x}} - \frac{8}{5x^3}$ (k) $\frac{4}{\sqrt{x}} - \frac{15}{x^6} + \frac{1}{2}$

(l) $-\frac{1}{2\sqrt{x^3}} - \frac{1}{\sqrt{x}} + x^2$ 3. (a) $\frac{3}{2}\sqrt{x} + \frac{1}{\sqrt{x}}$ (b) $4x^3 + 3x^2 - 1$ (c) $3x^2 + 1$ (d) $\frac{1}{x^2}$ (e) $\frac{1}{\sqrt{x^3}}$

(f) $\frac{1}{2} - \frac{1}{4\sqrt{x^3}}$ (g) -7 (h) $2x - \frac{8}{x^3}$ (i) $2x - \frac{2}{x^2} - \frac{4}{x^5}$ (j) $\frac{1}{2}\sqrt{\frac{3}{x}} + \frac{1}{6\sqrt{x^3}}$ (k) $2x - \frac{12}{5}\sqrt[5]{x} + \frac{2}{5^5\sqrt{x^3}}$

(l) $-\frac{3}{2\sqrt{x}}\left(\frac{1}{x} + 1\right)\left(\frac{1}{\sqrt{x}} - \sqrt{x}\right)^2$

EXERCISE 19.2.1

1. $m_{PQ} = 4 + h$; $\lim_{h \rightarrow 0} m_{PQ} = 4$ 2. $P(1, 1), Q\left(1 + h, \frac{2}{2+h}\right)$; $m_{PQ} = -\frac{1}{2+h}$;

$$\lim_{h \rightarrow 0} m_{PQ} = -\frac{1}{2} \quad \mathbf{3.} \quad -12 \quad \mathbf{4.} \quad (\mathbf{a}) \ 3 \ (\mathbf{b}) \ -\frac{1}{4} \ (\mathbf{c}) \ 12 \ (\mathbf{d}) \ 4 \ (\mathbf{e}) \ 4 \ (\mathbf{f}) \ \frac{7}{6} \ (\mathbf{g}) \ -\frac{1}{12} \ (\mathbf{h}) \ \frac{53}{16} \quad \mathbf{5.} \quad \pm\sqrt[8]{3}$$

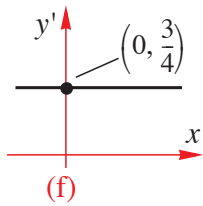
$$\mathbf{6.} \quad (\mathbf{a}) \ 2x - 12 \ (\mathbf{b}) \ -18 \ (\mathbf{c}) \ (8, -32) \quad \mathbf{7.} \quad (\mathbf{a}) \ -3x^2 + 3 \ (\mathbf{b}) \ 0 \ (\mathbf{c}) \ (\pm\sqrt{2}, \pm\sqrt{2}) \quad \mathbf{8.} \quad (\mathbf{a}) \ \left(\pm\frac{\sqrt{2}}{2}, -\frac{1}{16}\right), (0, 0)$$

$$(\mathbf{b}) \ \left\{x: \frac{-1}{\sqrt{2}} < x < 0\right\} \cup \left\{x: x > \frac{1}{\sqrt{2}}\right\} \quad \mathbf{9.} \quad x = \frac{1}{3}, -1 \quad \mathbf{10.} \quad (\mathbf{a}) \ -2, 6, 3 \ (\mathbf{b}) \ -2 \quad \mathbf{11.} \quad a = 1 \ b = -8$$

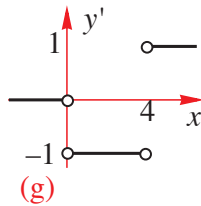
$$\mathbf{12.} \quad f'(a+b) = 2(a+b) = 2a+2b \quad \mathbf{13.} \quad (\mathbf{a}) \ 4a^2 - 2a, a \geq 0 \quad (\mathbf{b}) \ 4 - \frac{1}{a}, a > 0 \quad \mathbf{14.} \quad -56$$

EXERCISE 19.2.2

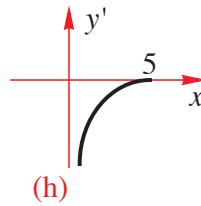
1. (a)



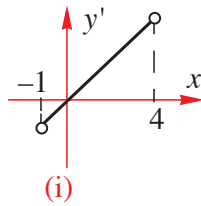
(b)



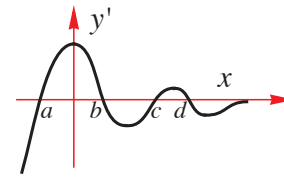
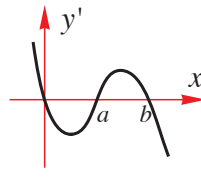
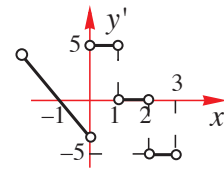
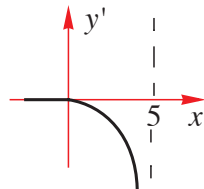
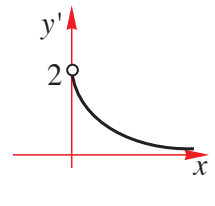
(c)



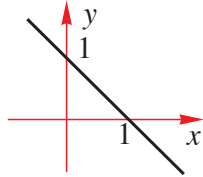
(d)



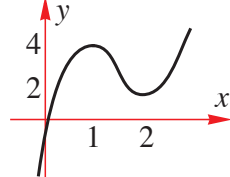
(e)



2.



3.



EXERCISE 19.2.3

$$\mathbf{1.} \quad (\mathbf{a}) \ 48t^3 - \frac{1}{2\sqrt{t}} \quad (\mathbf{b}) \ 2n - \frac{2}{n^2} - \frac{4}{n^5} \quad (\mathbf{c}) \ \frac{3}{2}\sqrt{r} + \frac{5}{6\sqrt{r}} - \frac{1}{\sqrt{r}} \quad (\mathbf{d}) \ 2\theta - \frac{9}{2}\sqrt{\theta} + 3 - \frac{1}{2\sqrt{\theta}} \quad (\mathbf{e}) \ 40 - 3L^2$$

$$(\mathbf{f}) \ -\frac{100}{v^3} - 1 \quad (\mathbf{g}) \ 6l^2 + 5 \quad (\mathbf{h}) \ 2\pi + 8h \quad (\mathbf{i}) \ 4n^3 - \frac{1}{3\sqrt[3]{n^2}} + \pi$$

$$\mathbf{2.} \quad (\mathbf{a}) \ \frac{8}{3t^3} \quad (\mathbf{b}) \ 2\pi r - \frac{20}{r^2} \quad (\mathbf{c}) \ \frac{5}{2}s^{3/2} + \frac{3}{s^2} \quad (\mathbf{d}) \ -\frac{6}{t^4} + \frac{2}{t^3} - \frac{1}{t^2} \quad (\mathbf{e}) \ -\frac{4}{b^2} + \frac{1}{2b^{3/2}} \quad (\mathbf{f}) \ 3m^2 - 4m - 4$$

EXERCISE 19.3

$$\mathbf{1.} \quad (\mathbf{a}) \ 3x^2 - 5x^4 + 2x + 2 \quad (\mathbf{b}) \ 6x^5 + 10x^4 + 4x^3 - 3x^2 - 2x \quad (\mathbf{c}) \ -\frac{4}{x^5} \quad (\mathbf{d}) \ 6x^5 + 8x^3 + 2x$$

$$\mathbf{2.} \quad (\mathbf{a}) \ -\frac{2}{(x-1)^2} \quad (\mathbf{b}) \ \frac{1}{(x+1)^2} \quad (\mathbf{c}) \ \frac{1-x^2-2x}{(x^2+1)^2} \quad (\mathbf{d}) \ \frac{-(x^4+3x^2+2x)}{(x^3-1)^2} \quad (\mathbf{e}) \ \frac{2x^2+2x}{(2x+1)^2} \quad (\mathbf{f}) \ \frac{1}{(1-2x)^2}$$

$$\mathbf{3.} \quad (\mathbf{a}) \ (\sin x + \cos x)e^x \quad (\mathbf{b}) \ \ln x + 1 \quad (\mathbf{c}) \ e^x(2x^3 + 6x^2 + 4x + 4) \quad (\mathbf{d}) \ 4x^3 \cos x - x^4 \sin x$$

$$(\mathbf{e}) \ -\sin^2 x + \cos^2 x \quad (\mathbf{f}) \ 2x \tan x + (1+x^2)\sec^2 x \quad (\mathbf{g}) \ \frac{4}{x^3}(x \cos x - 2 \sin x)$$

$$(\mathbf{h}) \ e^x(x \cos x + x \sin x + \sin x) \quad (\mathbf{i}) \ (\ln x + 1 + x \ln x)e^x$$

- 4.** (a) $\frac{\sin x - x \cos x}{\sin^2 x}$ (b) $\frac{-[\sin x(x+1) + \cos x]}{(x+1)^2}$ (c) $\frac{e^x}{(e^x+1)^2}$ (d) $\frac{2x \cos x - \sin x}{2x\sqrt{x}}$ (e) $\frac{\ln x - 1}{(\ln x)^2}$
 (f) $\frac{(x+1) - x \ln x}{x(x+1)^2}$ (g) $\frac{x e^x + 1}{(x+1)^2}$ (h) $\frac{-2}{(\sin x - \cos x)^2}$ (i) $\frac{x^2 - x + 2x \ln x}{(x + \ln x)^2}$ **5.** (a) $-5e^{-5x} + 1$
 (b) $4 \cos 4x + 3 \sin 6x$ (c) $-\frac{1}{3}e^{-\frac{1}{3}x} - \frac{1}{x} + 18x$ (d) $25 \cos 5x + 6e^{2x}$ (e) $4 \sec^2 4x + 2e^{2x}$
 (f) $-4 \sin(4x) + 3e^{-3x}$ (g) $\frac{4}{4x+1} - 1$ (h) 0 (i) $\frac{1}{2} \cos\left(\frac{x}{2}\right) - 2 \sin 2x$ (j) $7 \cos(7x-2)$ (k) $\frac{1}{2\sqrt{x}} - \frac{1}{x}$
 (l) $\frac{1}{x} + 6 \sin 6x$ **6.** (a) $2x \cos x^2 + 2 \sin x \cos x$ (b) $2 \sec^2 2\theta - \frac{\cos \theta}{\sin^2 \theta}$ (c) $\frac{1}{2\sqrt{x}} \cos \sqrt{x}$ (d) $\frac{1}{x^2} \sin\left(\frac{1}{x}\right)$
 (e) $-3 \sin \theta \cdot \cos^2 \theta$ (f) $e^x \cos(e^x)$ (g) $\frac{1}{x} \sec^2(\log_e x)$ (h) $\frac{-\sin 2x}{\sqrt{\cos 2x}}$ (i) $-\cos \theta \cdot \sin(\sin \theta)$
 (j) $4 \sin \theta \cdot \sec^2 \theta$ (k) $-5 \cos 5x \cdot \csc^2(5x)$ (l) $-6 \csc^2(2x)$
7. (a) $2e^{2x+1}$ (b) $-6e^{4-3x}$ (c) $-12xe^{4-3x^2}$ (d) $\frac{1}{2}\sqrt{e^x}$ (e) $\frac{1}{2\sqrt{x}}e^{\sqrt{x}}$ (f) e^{2x+4} (g) $2xe^{2x^2+4}$
 (h) $-\frac{6}{e^{3x+1}}$ (i) $(6x-6)e^{3x^2-6x+1}$ (j) $\cos(\theta)e^{\sin \theta}$ (k) $2 \sin(2\theta)e^{-\cos 2\theta}$ (l) $2x$ (m) $\frac{2e^{-x}}{(e^{-x}+1)^2}$
 (n) $3(e^x + e^{-x})(e^x - e^{-x})^2$ (o) e^{x+2} (p) $(-2x+9)e^{-x^2+9x-2}$ **8.** (a) $\frac{2x}{x^2+1}$ (b) $\frac{\cos \theta + 1}{\sin \theta + \theta}$
 (c) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$ (d) $-\frac{1}{x+1}$ (e) $\frac{3}{x}(\ln x)^2$ (f) $\frac{1}{2x\sqrt{\ln x}}$ (g) $\frac{1}{2(x-1)}$ (h) $\frac{-3x^2}{1-x^3}$ (i) $-\frac{1}{2(x+2)}$
 (j) $\frac{-2 \sin x \cos x}{\cos^2 x + 1}$ (k) $\frac{1}{x} + \cot x$ (l) $\frac{1}{x} + \tan x$
9. (a) $\ln(x^3+2) + \frac{3x^3}{x^3+2}$ (b) $\frac{\sin^2 x}{2\sqrt{x}} + 2\sqrt{x} \sin x \cos x$ (c) $-\frac{1}{\sqrt{\theta}} \sin \sqrt{\theta} \cdot \cos \sqrt{\theta}$
 (d) $(3x^2 - 4x^4)e^{-2x^2+3}$ (e) $-(\ln x + 1) \sin(x \ln x)$ (f) $\frac{1}{x \ln x}$
 (g) $\frac{(2x-4) \cdot \sin(x^2) - 2x \cdot \cos(x^2)(x^2-4x)}{(\sin x^2)^2}$ (h) $\frac{10(\ln(10x+1)-1)}{[\ln(10x+1)]^2}$
 (i) $(\cos 2x - 2 \sin 2x)e^{x-1}$ (j) $2x \ln(\sin 4x) + 4x^2 \cot 4x$ (k) $(\cos \sqrt{x} - \sin \sqrt{x}) \frac{1}{2\sqrt{x}} e^{-\sqrt{x}}$
 (l) $-(2 \sin x + 2x \cos x) \cdot \sin(2x \sin x)$ (m) $\frac{e^{5x+2}(9-20x)}{(1-4x)^2}$ (n) $\frac{\cos^2 \theta + \sin^2 \theta \ln(\sin \theta)}{\sin \theta \cos^2 \theta}$
 (o) $\frac{x+2}{2(x+1)\sqrt{x+1}}$ (p) $\frac{2x^2+2}{\sqrt{x^2+2}}$ (q) $\frac{10x^3+9x^2+4x+3}{3(x+1)^{2/3}}$ (r) $\frac{3x^2(3x^3+1)}{2\sqrt{x^3+1}}$
 (s) $\frac{2}{x^2+1} - \frac{1}{x^2} \ln(x^2+1)$ (t) $\frac{2}{x(x+2)}$ (u) $\frac{2-x}{2x^2\sqrt{x-1}}$ (v) $\frac{-x^2+x-9}{\sqrt{x^2+9}} \cdot e^{-x}$ (w) $\frac{7x^3-12x^2-8}{2\sqrt{2-x}}$
 (x) $nx^{n-1} \ln(x^n-1) + \frac{nx^{2n-1}}{x^n-1}$ **10.** $x=1$ **11.** 0 **12.** 0 **13.** 1 **14.** $-2e$ **15.** (a) $\cos^2 x - \sin^2 x$
 (b) $\frac{\pi}{180} \cos x^\circ$ (c) $-\frac{\pi}{180} \sin x^\circ$ **16.** (b) i. $2x \sin x \cos x + x^2 \cos^2 x - x^2 \sin^2 x$

ii. $e^{-x^3}(2 \cos 2x \ln \cos x - 3x^2 \sin 2x \ln \cos x - \sin 2x \tan x)$ **17.** (a) i. $-\frac{3}{x}(\ln x)^2$ ii. $-\frac{3x^2}{1-x^3}$

(b) i. $-2e^{-2x} \cdot \cos(e^{-2x})$ ii. $-2x \cos x^2 \cdot e^{-\sin x^2}$ **18.** $-\frac{1}{5}k$ **19.** $x = a, b, \frac{mb + na}{m + n}$

20. $\{\theta: n \tan \theta^m \cdot \tan \theta^n = m \theta^{m-n}\}$ **21.** (a) $-4 \csc(4x)$ (b) $2 \sec(2x) \tan(2x)$

(c) $3 \cot(3x) \csc(3x)$ (d) $-3 \sin(3x)$ (e) $\csc^2\left(\frac{\pi}{4} - x\right)$ (f) $-2 \sec(2x) \tan(2x)$

22. (a) $2x \sec(x^2) \tan(x^2)$ (b) $\sec^2 x$ (c) $\tan x$ (d) $-3 \cot^2 x \csc^2 x$ (e) $x \cos x + \sin x$

(f) $-2 \cot x \csc^2 x$ (g) $4x^3 \csc(4x) - 4x^4 \cot(4x) \csc(4x)$ (h) $2 \cot x \sec^2(2x) - \csc^2 x \tan(2x)$

(i) $\frac{\sec x \tan x - \sin x}{2\sqrt{\cos x + \sec x}}$ **23.** (a) $e^{\sec x} \sec x \tan x$ (b) $e^x \sec(e^x) \tan(e^x)$

(c) $e^x \sec(x) + e^x \sec(x) \tan(x)$ (d) $\frac{-\csc^2(\log x)}{x}$ (e) $-5 \csc(5x) \sec(5x)$

(f) $\frac{\cot(x)}{x} - \csc^2(x) \log x$ (g) $-\cos x \cot(\sin x) \csc(\sin x)$ (h) $-\cos(\csc x) \cot x \csc x$ (i) 0

EXERCISE 19.4

1. (a) $\frac{2}{4x^2 + 1}$ (b) $\frac{1}{\sqrt{9 - x^2}}$ (c) $\frac{-2}{\sqrt{1 - 4x^2}}$ (d) $\frac{4}{\sqrt{1 - 16x^2}}$ (e) $\frac{2}{x^2 + 4}$ (f) $\frac{1}{\sqrt{2x - x^2}}$ (g) $\frac{-1}{\sqrt{16 - x^2}}$

(h) $\frac{1}{\sqrt{4 - (x+1)^2}}$ (i) $\frac{1}{(4-x)^2 + 1}$ (j) $\frac{-1}{\sqrt{4x - x^2}}$ (k) $\frac{6}{4x^2 + 9}$ (l) $\frac{-1}{\sqrt{-x^2 + x + 2}}$

2. (a) $\frac{2x}{x^4 + 1}$ (b) $\frac{1}{2\sqrt{x - x^2}}$ (c) $\frac{1}{2\sqrt{x^3 - x^2}}$ (d) $\frac{-\sin x}{\sqrt{1 - \cos^2 x}} = \begin{cases} -1 & \text{if } \sin x > 0 \\ 1 & \text{if } \sin x < 0 \end{cases}$ (e) $\frac{1}{2x\sqrt{x-1}}$

(f) $\frac{1}{\sqrt{1-x^2} \text{Sin}^{-1} x}$ (g) $\frac{e^x}{1+e^{2x}}$ (h) $\frac{1}{\sqrt{e^{2x}-1}}$ (i) $\frac{e^{\arcsin x}}{\sqrt{1-x^2}}$ (j) $\frac{-4}{(4x^2+1)[\tan^{-1}(2x)]^2}$

(k) $\frac{-1}{\sqrt{1-x^2}(\sin^{-1}(x))^{3/2}}$ (l) $\frac{2}{\sqrt{1-x^2}(\cos^{-1}(x))^3}$ (m) $\frac{-4x}{\sqrt{1-4x^2}}$ (n) $\frac{-4x}{\sqrt{1-4x^2}}$ (o) $\frac{-1}{x^2\sqrt{1-x^2}}$

3. (a) $\tan^{-1} x + \frac{x}{1+x^2}$ (b) $\frac{x - \sqrt{1-x^2} \sin^{-1} x}{x^2 \sqrt{1-x^2}}$ (c) $\frac{x + \sqrt{1-x^2} \cos^{-1} x}{(\cos^{-1} x)^2 \sqrt{1-x^2}}$

(d) $\frac{-2x^2 \tan^{-1} x + x - 2 \tan^{-1} x}{x^3(x^2+1)}$ (e) $\frac{2x^2 \log x + \sqrt{1-x^4} \sin^{-1}(x^2)}{x\sqrt{1-x^4}}$ (f) $\frac{-\sqrt{1-x} \cos^{-1} \sqrt{x} - \sqrt{x}}{2x^{3/2} \sqrt{1-x}}$

(g) $e^x \tan^{-1}(e^x) + \frac{e^{2x}}{1+e^{2x}}$ (h) $2x \tan^{-1}\left(\frac{x}{2}\right) + 2$ (i) $1 - \frac{x}{\sqrt{4-x^2}} \text{Sin}^{-1}\left(\frac{x}{2}\right)$ **4.** $0, k = \frac{\pi}{2}$

6. (b) $k = \frac{\pi}{2}$

7. (a) $f'(x) = \frac{-\pi}{x\sqrt{x^2-\pi^2}}, x > \pi$ and $\frac{\pi}{x\sqrt{x^2-\pi^2}}, x < -\pi$; $\text{dom}(f) =]-\infty, -\pi[\cup]\pi, \infty[$

(b) $f'(x) = \frac{1}{x\sqrt{2x-1}}, x > \frac{1}{2}$; $\text{dom}(f') = \left] \frac{1}{2}, \infty \right[$, $\text{dom}(f) = \left[\frac{1}{2}, \infty \right[$

(c) $f'(x) = \frac{1}{\sqrt{1-x^2}} \text{Cos}^{-1}\left(\frac{x}{2}\right) - \frac{1}{\sqrt{4-x^2}} \text{Sin}^{-1}(x), -1 < x < 1$; $\text{dom}(f) = [-1, 1]$

(d) $f'(x) = -\frac{2}{x^2+1}, x > 0$ and $\frac{2}{x^2+1}, x < 0$; $\text{dom}(f) =]-\infty, \infty[$

(e) $f'(x) = \frac{a}{\sqrt{1-a^2x^2}}, |x| < \frac{1}{a}$; $\text{dom}(f) = \left[-\frac{1}{a}, \frac{1}{a}\right]$

(f) $f'(x) = \frac{2}{\sqrt{1-x^2}}, -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ and $f'(x) = \frac{-2}{\sqrt{1-x^2}}, -1 < x < -\frac{1}{\sqrt{2}}$ or $\frac{1}{\sqrt{2}} < x < 1$;
 $\text{dom}(f) = [-1, 1]$

(g) $f'(x) = \frac{2}{x^2+1}, x > 0$ and $\frac{-2}{x^2+1}, x < 0$; $\text{dom}(f) =]-\infty, \infty[$

(h) $f'(x) = \frac{2}{x^2+1}, |x| < 1$ and $\frac{-2}{x^2+1}, |x| > 1$; $\text{dom}(f) =]-\infty, \infty[$

8. (a) $\frac{nx^{n-1}}{1+x^{2n}} + \frac{n}{1+x^2}(\arctan x)^{n-1}$ (b) 0 (c) $2\sqrt{1-x^2}$ (d) $\frac{1}{2\sqrt{(a-x)(x-b)}}$ (e) $\frac{1}{2(1+x^2)}$

(f) $-\frac{1}{x^2+1}$ **9.** (a) $x \geq 0$ (b) $\frac{-1}{(x+1)\sqrt{x}}, x > 0$

EXERCISE 19.5

1. (a) $(\ln 4)4^x$ (b) $(\ln 3)3^x$ (c) $(\ln 8)8^x$ (d) $3(\ln 5)5^x$ (e) $7(\ln 6)6^x$ (f) $2(\ln 10)10^x$

(g) $(\ln 6)6^{x-2}$ (h) $3(\ln 2)2^{3x+1}$ (i) $-5(\ln 7)7^{3-x}$ **2.** (a) $x(\ln 3)3^x + 3^x$

(b) $4\cos(2x)2^x + 2\ln(2)\sin(2x)2^x$ (c) $\ln(5)5^x e^{-x} - 5^x e^{-x}$ (d) $2x8^{-x} - \ln(8)8^{-x}x^2$

(e) $\frac{(1+4^x) - (x+2)\ln(4)4^x}{(1+4^x)^2}$ (f) $-\sin x 5^x + \ln(5)5^x \cos x$ **3.** (a) $\frac{1}{(\ln 5)x}$ (b) $\frac{1}{(\ln 10)x}$

(c) $\frac{1}{(\ln 4)x}$ (d) $\frac{1}{(\ln 9)(x+1)}$ (e) $\frac{2x}{(\ln 2)(x^2+1)}$ (f) $\frac{1}{2(\ln 5)(x-5)}$ (g) $\log_2 x + \frac{1}{\ln 2}$

(h) $(\ln 3)3^x \log_3 x + \frac{3^x}{(\ln 3)x}$ (i) $(\ln a)a^x \log_a x + \frac{a^x}{(\ln a)x}$ (j) $\frac{(\ln a)^2 x a^x \log_a x - a^x}{(\ln a)x(\log_a x)^2}$

(k) $\frac{(\ln(10))\log_{10}(x+1) - 1}{\ln(10)(\log_{10}(x+1))^2}$ (l) $\frac{(\ln(2))2\log_2 x - 2}{\ln(2)(\log_2 x)^2}$ **4.** $\frac{1}{\ln 2}$ **5.** $0, -\frac{2}{\ln 2}$ **6.** $\frac{1 - \ln 3}{3}$

7. $\pi 2^{-\pi} \sqrt{3} + \frac{\sqrt{3}\pi \ln \pi}{2}$ **8.** 1.25 **9.** (a) $20 + 10 \ln 10$ (b) $(\ln 4)\cos(1)$ (c) $\frac{1}{2}$ (d) $10 - \frac{10}{\ln 10}$

10. (a) $4 \times 5^{4x+1} \ln 5$ (b) $3^{x-x^3}(1-3x^2)\ln 3$ (c) $2(10^{2x-3})\ln 10$ (d) $9^{\sqrt{x}-x} \left(\frac{1}{2\sqrt{x}} - 1 \right) \ln 9$

(e) $-2^{\cos(2x)+1} \ln 2 \sin 2x$ (f) $\frac{-4^{\sqrt{\cos 2x}} \ln 4 \sin 2x}{\sqrt{\cos 2x}}$ (g) $2^x \cos 2^x \ln 2$ (h) $2^{\sin x} \cos x \ln 2$

(i) $-7^{\left(\frac{1}{x}-2x\right)} (2+x^{-2}) \ln 7$ **11.** (a) $\frac{2 \cot 2x}{\ln 2}$ (b) $\frac{x}{(x^2-1)\ln 5}$ (c) $\frac{1}{2(\sqrt{x}-10)\sqrt{x} \ln 10}$

(d) $\frac{-4 \sec^2 2x}{\ln 2(4-2 \tan 2x)}$ (e) $\frac{1}{2\sqrt{x-x^2} \sin^{-1} \sqrt{x} \ln 2}$ (f) $\frac{-1}{((1-x^2)+1) \tan^{-1}(1-x) \ln 3}$

(g) $\frac{3x^2}{(x^3-3)\ln 3}$ (h) $\frac{-1}{2(2-x)\ln 2}$ (i) $-\frac{1}{2\ln 10}\tan\left(\frac{x}{2}-2\right)$ **12.** (a) $x^x(\ln x + 1)$
 (b) $x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$ (c) $(1 - \ln x)x^{\frac{1}{x}-2}$ (d) $2\ln(x)x^{\ln x - 1}$

EXERCISE 19.6

1. i. $20x^3$ ii. $48(1+2x)^2$ iii. $\frac{2}{x^3}$ iv. $\frac{2}{(1+x)^3}$ v. 2 vi. $\frac{6}{(x-2)^3}$ vii. $\frac{42}{x^8}$
 viii. $24(1-2x)$ ix. $-\frac{1}{x^2}$ x. $-\frac{2(x^2+1)}{(1-x^2)^2}$ xi. $-16\sin 4\theta$ xii. $2\cos x - x\sin x$
 xiii. $6x^2\cos x + 6x\sin x - x^3\sin x$ xiv. $\frac{1}{x}$ xv. $\frac{10}{(2x+3)^3}$ xvi. $6xe^{2x} + 12x^2e^{2x} + 4x^3e^{2x}$
 xvii. $\frac{8\sin 4x - 15\cos 4x}{e^x}$ xviii. $2\cos x^2 - 4x^2\sin x^2$ xix. $\frac{-48(x^2+2x^5)}{(4x^3-1)^3}$ xx. $\frac{10}{(x-3)^3}$
2. (a) $\frac{-2x}{(x^2+1)^2}$ (b) $\frac{x}{(1-x^2)^{3/2}}$ (c) $\frac{-x}{(1-x^2)^{3/2}}$ (d) $\frac{2}{(x^2+1)^2}$ (e) $\frac{2x-1}{4(x-x^2)^{3/2}}$ (f) $\frac{2x-3x^2}{4\sqrt{(x^3-x^2)^3}}$
 (g) $\frac{2\arctan(x)}{x^3} - \frac{1}{x^2(1+x^2)} - \frac{1+3x^2}{(x+x^3)^2}$ (h) $\frac{1}{\sqrt{4x-x^3}} + \frac{x^{3/2}}{2(4-x^2)^{3/2}} - \frac{1}{4}\left[\text{Sin}^{-1}\left(\frac{x}{2}\right)\right]x^{-3/2}$
 (i) $\frac{e^x(-x+2e^{2x}-2)}{(e^{2x}-1)\sqrt{1-e^{2x}}}$ **3.** $\frac{6\ln x - 5}{x^4}, \frac{n^2\ln x + n\ln x - 2n - 1}{x^{n+2}}$
4. $f'(x) = -\frac{1}{(x+1)^2}, f''(x) = \frac{2}{(x+1)^3}, f^{(iii)}(x) = -\frac{6}{(x+1)^4}, f^{(iv)}(x) = \frac{24}{(x+1)^5}$
 $f^{(v)}(x) = -\frac{120}{(x+1)^6}, \dots, f^{(n)}(x) = (-1)^n \frac{n!}{(x+1)^{n+1}}$
5. $f(x) = \left(\frac{x+1}{x-1}\right)^n \Rightarrow f'(x) = \frac{4n(n+x)}{(x^2-1)^2}\left(\frac{x+1}{x-1}\right)^n$ **7.** (a) $a^n e^{ax}$ (b) $\frac{(-1)^n 2^n n!}{(2x+1)^{n+1}}$
 (c) $n = 2k: y^{(n)}(x) = (-1)^k a^{2k} \sin(ax+b), k = 1, 2, \dots$ **8.** (a) $2 + \frac{1}{8\sqrt{2}}$ (b) $\frac{3+\pi}{2}$
9. -1 **10.** $[0, 1.0768] \cup [3.6436, 2\pi]$

EXERCISE 19.7.1

1. (a) $-2x$ (b) $-\frac{x}{y}$ (c) $\frac{1}{x^3 y}$ (d) $-\frac{y}{x+1}$ (e) $-\frac{ye^x}{1+e^x}$ (f) $\frac{\sin x - y}{x}$ (g) $-y$ (h) $\frac{1-3x^4 y}{x^5}$
 (i) $-\frac{y\cos x + 2}{\sin x}$ (j) -1 (k) $\frac{4x^3}{3y^2+1}$ (l) $\sqrt{x+y}-1$ **2.** $(1, 5), 0$
4. $\left(\frac{3-2\sqrt{10.6}}{2}, \frac{80+4\sqrt{265}}{40}\right), \left(\frac{3+2\sqrt{10.6}}{2}, \frac{80-4\sqrt{265}}{40}\right)$ **5.** (a) $y = \frac{x \pm \sqrt{5x^2-80}}{2}$
 (c) $\frac{dy}{dx} = \frac{2x+y}{2y-x}$ (d) $\frac{5x \pm \sqrt{5x^2-80}}{2\sqrt{5x^2-80}}$ (e) Hyperbola. **6.** (a) Dom = Ran = $[-2, 2]$ (b) $-\frac{x^3}{y^3}$
 (c) $-\frac{x^3}{(\sqrt[4]{16-x^4})^3}$ (d) small (e) dom = ran = $[-k, k]$ (f) $\frac{dy}{dx} = -\frac{x^{2n-1}}{y^{2n-1}}$ **7.** (a) $\frac{-v}{p\gamma}$

- (b) $\frac{n(m-1)x^{m-2}}{m(n+1)y^n}$ **8.** (a) $\frac{1}{11}$ (b) -1 **9.** (a) $\frac{y}{xy-x}$ (b) $\frac{(1+y^2)(\tan^{-1}y-1)}{1-x+y^2}$
10. (a) undefined (b) At (0.8042, 0.5), grad = 1.32; at (0.0646, 0.5), grad = 3.74

EXERCISE 19.7.2

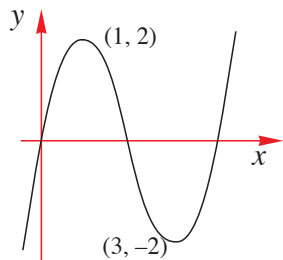
- 1.** i. $\frac{dy}{dx} = -\frac{5}{x^2}, \frac{d^2y}{dx^2} = \frac{10}{x^3}$ ii. $\frac{dy}{dx} = \frac{2x^3+18x^2}{(x+6)^2}, \frac{d^2y}{dx^2} = \frac{2x^3+36x^2+216x}{(x+6)^3}$
 iii. $\frac{dy}{dx} = -\frac{x}{y}, \frac{d^2y}{dx^2} = -\frac{4}{y^3}$ iv. $\frac{dy}{dx} = -\frac{1}{3e^{3y}}, \frac{d^2y}{dx^2} = -\frac{1}{3e^{6y}}$
 v. $\frac{dy}{dx} = \frac{2\cos 2x}{\cos y}, \frac{d^2y}{dx^2} = -\frac{4\sin 2x}{\cos y} + \frac{4\cos^2 2x \tan y}{\cos^2 y}$ vi. $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}, \frac{d^2y}{dx^2} = \frac{x+\sqrt{xy}}{2x^2}$
 vii. $\frac{dy}{dx} = \frac{x+3y}{3x+y}, \frac{d^2y}{dx^2} = \frac{8x^2+48xy+8y^2}{(3x+y)^3}$ viii. $\frac{dy}{dx} = -\frac{y}{x}, \frac{d^2y}{dx^2} = \frac{2y}{x^2}$
2. ii. -3 iii. 2 (iv) **3.** ii. -1 iii. 10 iv. **4.** i. Dom $[0,6]$ Ran $[-4,2]$
 ii. $\frac{dy}{dx} = \frac{3-x}{y+1}, \frac{d^2y}{dx^2} = -\frac{9}{(y+1)^3}$ **5.** $\frac{7}{a}$

EXERCISE 20.1

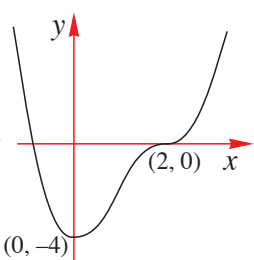
- 1.** (a) $y = 7x - 10$ (b) $y = -4x + 4$ (c) $4y = x + 5$ (d) $16y = -x + 21$ (e) $4y = x + 1$ (f) $4y = x + 2$
 (g) $y = 28x - 48$ (h) $y = 4$ **2.** (a) $7y = -x + 30$ (b) $4y = x - 1$ (c) $y = -4x + 14$ (d) $y = 16x - 79$
 (e) $2y = 9 - 8x$ (f) $y = -4x + 9$ (g) $28y = -x + 226$ (h) $x = 2$ **3.** (a) $y = 2e^x - e$ (b) $y = e$ (c) $y = \pi$
 (d) $y = -x$ (e) $y = x$ (f) $ey = (2e-1)x - e^2 + 2e - 1$ (g) $y = ex$ (h) $y = 2x + 1$
4. (a) $2ey = -x + 2e^2 + 1$ (b) $x = 1$ (c) $x = \pi$ (d) $y = x - 2\pi$ (e) $y = -x + \pi$
 (f) $(2e-1)y = -ex + 3e^2 - 4e + 1$ (g) $ey = -x$ (h) $2y = -x + 2$ **5.** A: $y = 28x - 44$,
 B: $y = -28x - 44$, Isosceles. $z \equiv (0, a^2 - 3a^4)$ **6.** 2 sq. units, $y = 2x - 1$ **7.** $4y = 3x$
8. $by = \sqrt{a^2 - b^2}x$ **9.** $y = 4x - 9$ **10.** $y = \log_6 4$ **11.** $8y = 4(\pi + 2)x - \pi^2$;
 $4(\pi + 2)y = -8x + 4\pi + \pi^2$ **12.** A: $y = -8x + 32$, B: $y = 6x + 25, \left(\frac{1}{2}, 28\right)$ **13.** $y = -x$, Tangents:
 $y = \frac{1}{2}, y = -\frac{1}{2}$ $\left(-\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, -\frac{1}{2}\right)$ tangent and normal meet at $(0.5, -0.5)$
14. (a) $y = 3x - 7$ (b) $Q \equiv (2, -1)$ **15.** $m = -2, n = 5$ **16.** (a) $y = 4x - 2$ (b) $37y = 26x + 70$
 (c) $16y = x + 65$ (d) $y = \frac{4}{\pi} + \frac{\pi^2}{4(\pi-2)} - \frac{\pi^2}{4(\pi-2)}x$ (e) $5y = 6x - 1$
17. (a) $y = 1$ (b) At $(1, 2)$ $y = 2$; At $(-1, -2)$ $y = -2$ **18.** (a) $l_1: 3y = -2x + 1, l_2: 2y = 3x + 5$
 (b) $l_1: 2y = x, l_2: y = -2x + 5$ (c) $l_1: 6y = x + 16, l_2: y = -6x + 15$ **19.** $\left(\frac{2}{3}, 1\right)$

EXERCISE 20.2

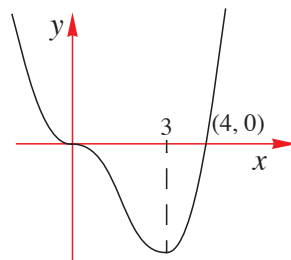
1. (a)



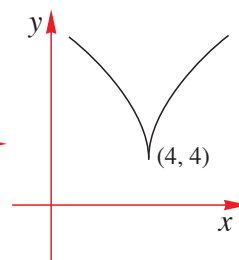
(b)



(c)



(d)



2. (a) max at (1, 4) (b) min at $(-\frac{9}{2}, -\frac{81}{4})$ (c) min at (3, -45) max (-3, 63) (d) max at (0, 8),

min at (4, -24) (e) max at (1, 8), min at (-3, -24) (f) min at $(\frac{1+\sqrt{13}}{3}, \frac{70-26\sqrt{13}}{27})$,

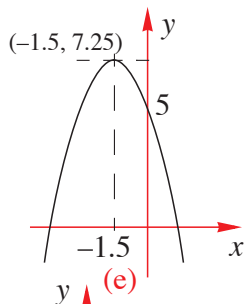
max at $(\frac{1-\sqrt{13}}{3}, \frac{70+26\sqrt{13}}{27})$ (g) min at (1, -1) (h) max at (0, 16), min at (2, 0),

min at (-2, 0) (i) min at (1, 0) max at $(-\frac{1}{3}, \frac{32}{27})$ (j) min at $(\frac{4}{9}, -\frac{4}{27})$ (k) min at (2, 4),

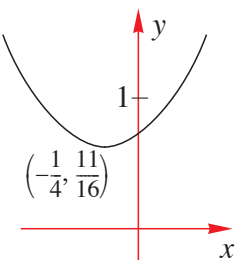
max at (-2, -4) (l) min at (1, 2), min at (-1, 2)

3.

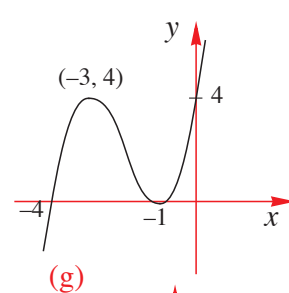
(a)



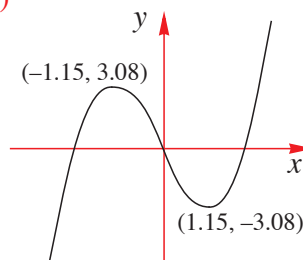
(b)



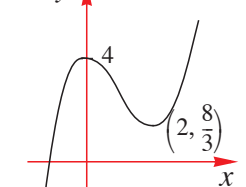
(c)



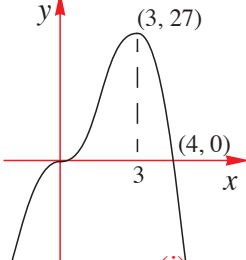
(d)



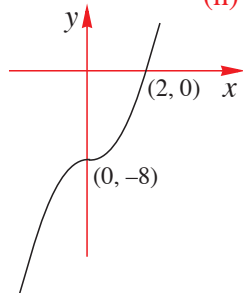
(e)



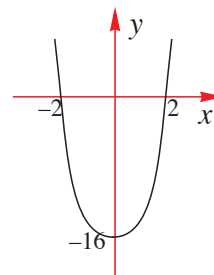
(f)



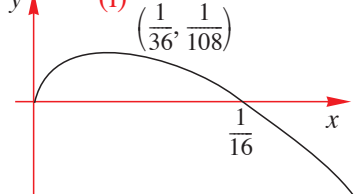
(g)



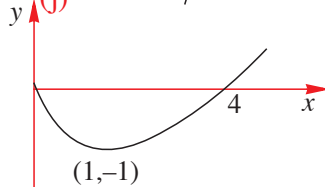
(h)



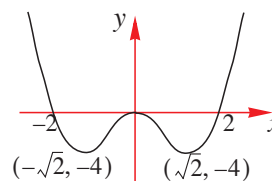
(i)



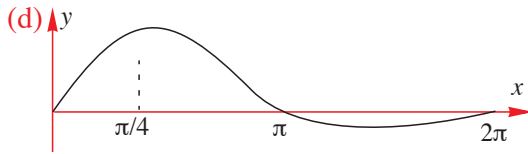
(j)



4. min at (1, -3), max at (-3, 29), non-stationary infl (-1, 13) 5.

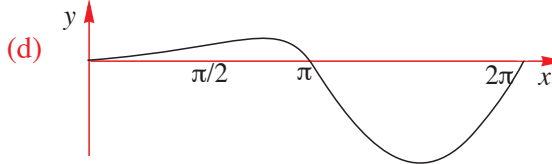


6. (a) i. $(\cos x - \sin x)e^{-x}$ ii. $-2\cos x \cdot e^{-x}$ (b) i. $\frac{\pi}{4}, \frac{5\pi}{4}$ ii. $\frac{\pi}{2}, \frac{3\pi}{2}$ (c) Inf. $(\frac{\pi}{2}, e^{-\frac{\pi}{2}})$ $(\frac{3\pi}{2}, -e^{-\frac{3\pi}{2}})$



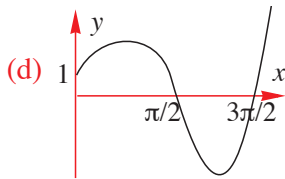
7. (a) i. $e^x(\sin x + \cos x)$ ii. $2e^x \cos x$ (b) i. $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

ii. $x = \frac{\pi}{2}, \frac{3\pi}{2}$ (c) st. pts. $\left(\frac{3\pi}{4}, \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}\right), \left(\frac{7\pi}{4}, -\frac{1}{\sqrt{2}}e^{\frac{7\pi}{4}}\right)$ Infl. pts. $\left(\frac{\pi}{2}, e^{\frac{\pi}{2}}\right), \left(\frac{3\pi}{2}, -e^{\frac{3\pi}{2}}\right)$



8. (a) i. $e^x(\cos x - \sin x)$ ii. $-2\sin x \cdot e^x$ (b) i. $\frac{\pi}{4}, \frac{5\pi}{4}$

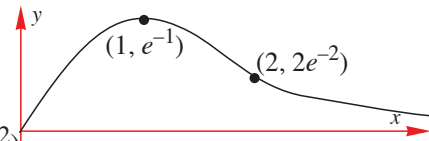
ii. $0, \pi, 2\pi$ (c) st. pts. $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}e^{\frac{\pi}{4}}\right), \left(\frac{5\pi}{4}, -\frac{1}{\sqrt{2}}e^{\frac{5\pi}{4}}\right)$ Inf. pts. $(0, 1), (\pi, -e^\pi), (2\pi, e^{2\pi})$



9. (a) i. $(1-x)e^{-x}$ ii. $(x-2)e^{-x}$

(b) i. $x = 1$ ii. $x = 2$

(c) st. pt. $(1, e^{-1})$ Inf. pt. $(2, 2e^{-2})$



10. (a) 8 (b) 0 (c) 4 (d) $27\sqrt[3]{9} \approx 56.16$ **11.** i. min value -82 . ii. max value 26

12. (a) A i. Yes ii. non-stationary pt of inflect. B. i. Yes ii. Stationary point (local/global min) C. i. Yes ii. non-stationary pt of inflect.

(b) A. i. No ii. Local/global max B. i. No. ii. Local/global min C. i. Yes ii. Stationary point (local max)

(c) A. i. Yes ii. Stationary point (local/global max) B. i. Yes ii. Stationary point (local min)

C. i. Yes ii. non-stationary pt of inflect.

(d) A. i. Yes ii. Stationary pt (local/global max) B. i. No ii. Local min

C. i. Yes ii. Stationary point (local max)

(e) A. i. No ii. Cusp (local min) B. i. Yes ii. Stationary pt of inflect C. i. Yes ii. Stationary point (local max)

(f) A. i. Yes ii. Stationary point (local/global max) B. i. Yes ii. Stationary point (local/global min) C. i. No ii. Tangent parallel to y-axis.

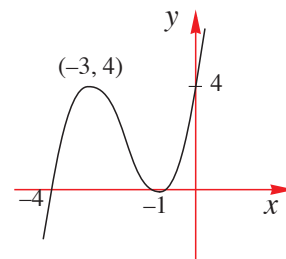
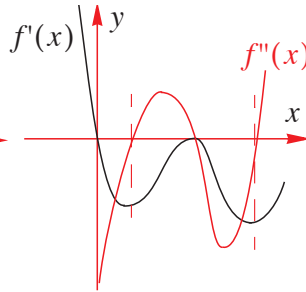
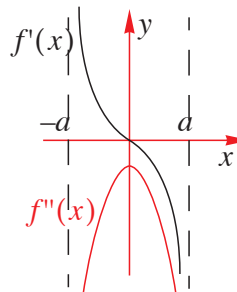
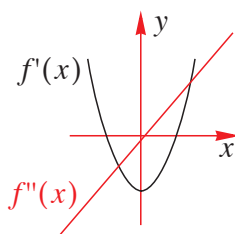
13. (a) i. A ii. B iii. C (b) i. C ii. B iii. A

14. i.

ii.

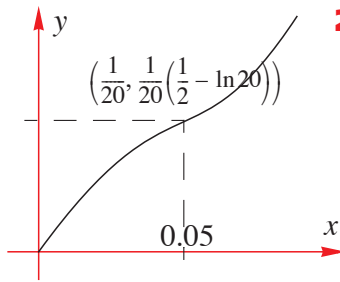
iii.

15. $y = x^3 + 6x^2 + 9x + 4$



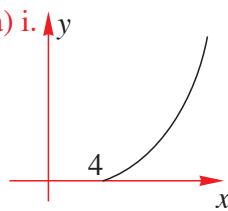
16. $f(x) = \frac{1}{3}x^3 - x^2 - 3x - 6$ **17.** $f(x) = 3x^5 - 20x^3$

18.

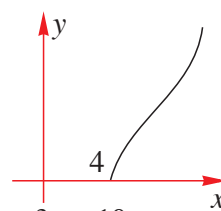


19. $m = -0.5, n = 1.5$

20. (a) i.



ii.



(b) i. $\frac{3}{2}\sqrt{x-4}$

ii. $\frac{3x-10}{2\sqrt{x-4}}$

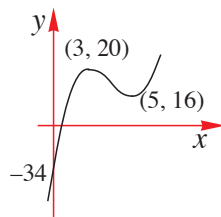
21. $a = 2, b = -3, c = 0$ 22. Stationary points: local min at $(-1, 0)$ and local max at $(1, 4e^{-1})$.
inflection. pts are: $(1 + \sqrt{2}, (6 + 4\sqrt{2})e^{-(1+\sqrt{2})})$ and $(1 - \sqrt{2}, (6 - 4\sqrt{2})e^{-(1-\sqrt{2})})$

23. Absolute min at $\sim \left(\frac{-3 + \sqrt{13}}{2}, -2.1733\right)$, Local max at $\sim \left(\frac{-3 - \sqrt{13}}{2}, 0.2062\right)$

Inflection pts at $\sim (-0.4384, -1.4489)$ and $(-4.5615, 0.1488)$

24. - 27. are left as questions for classroom discussion.

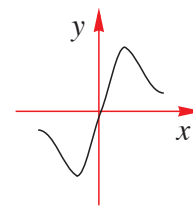
28. $a = 1, b = -12,$
 $c = 45, d = -34$



29. (b) $b = 1$

(c) $a = \frac{1}{\sqrt{2}}$

(d) $f(x) = \frac{1}{\sqrt{2}}xe^{-x^2}$



30. (a) 2.7983, 6.1212, 9.3179 (b) Make use of graphics calculator to verify your sketch.

EXERCISE 20.3

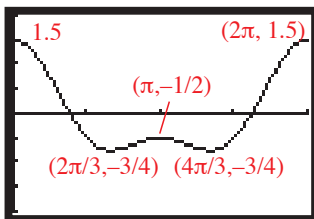
1. (a) L.min at $x = \frac{4}{\sqrt{3}}$. L.max at $x = -\frac{4}{\sqrt{3}}$ (b) L.max at $x = 0$, L.min at $x = \pm 1$

(c) L.max at $x = 0.25$ (d) L.max at $x = 1$ (e) none (f) L.max at $x = 0.5$, L.min at $x = 1, 0$

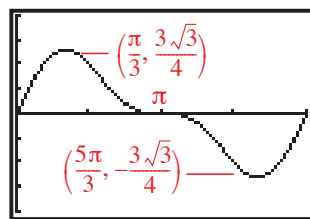
(g) L.max at $x = 1$, L.min at $x = -1$ (h) none 2. (a) max = 120, min = $-\frac{128}{3\sqrt{3}}$ (b) max = 224,

min = -1 (c) max = 0.5, min = 0 (d) max = 1, min = 0.

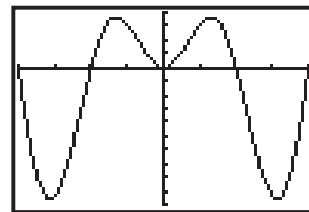
3.



4.

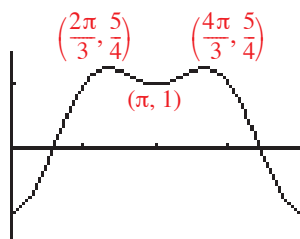


5. stationary points occur where $\tan x = -x$



6. (a) L.min at $(1, 2)$; Infl. pt at $\left(3, \sqrt{3} + \frac{1}{3}\sqrt{3}\right)$ (b) L.min at $(1, 2)$; L.max at $(-3, -6)$ (c) none

7.



For Q.8. - Q.11 verify your graphs with graphics calculator

8. (a) Global. min at $(0, 0)$; local max at $(2, 4e^{-2})$

Infl. pts $(2 - \sqrt{2}, (6 - 4\sqrt{2})e^{-(2-\sqrt{2})})$

$(2 + \sqrt{2}, (6 + 4\sqrt{2})e^{-(2+\sqrt{2})})$

(b) Global. max: $(0, e^4)$. Infl. pt: $\left(\pm \frac{1}{\sqrt{2}}, e^{3.5}\right)$ (c) L.max: $\left(-2, -\frac{1}{2}e\right)$

9. (a) Global max at (e, e^{-1}) . Infl. pt at $(e^{1.5}, 1.5e^{-1.5})$ (b) Global min at $(\frac{1}{\sqrt{2}}, 2 + \frac{1}{2}\ln 2)$

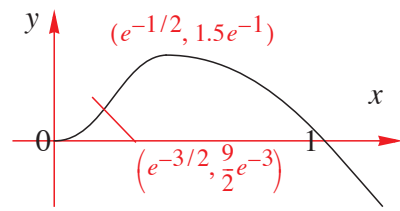
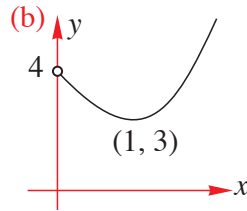
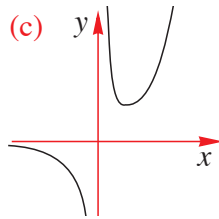
(c) Global min at $(2, 1 + \ln 2)$; Infl. pt at $(4, 2 + \ln 4)$ (d) none **10.** Global min at $(\frac{\pi}{6}, \sqrt{3})$

11. (a) $f'(x) = (x-2)^{a-1}(x+2)^{b-1}((a+b)x + 2(a-b))$

(b) i. $f(x) = \frac{x-2}{x+2}$; none ii. $f(x) = (x-2)^2(x+2)$ local max at $(-\frac{2}{3}, \frac{256}{27})$; local min at $(2, 0)$

iii. $f(x) = (x-2)^2(x+2)^2$; local min at $(\pm 2, 0)$, local max at $(0, 16)$.

12. (b) local min at $(c, 5^c + c^{-1})$ **13.** (a) Global. min at $(1, c-1)$; $c \geq 1$ **14.**



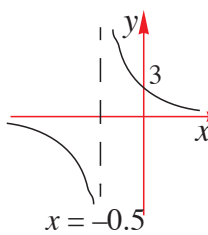
15. Global max at $(e^{0.5}, 0.5e^{-1})$; Infl. pt at $(e^{5/6}, \frac{5}{6}e^{-5/3})$.

EXERCISE 20.4

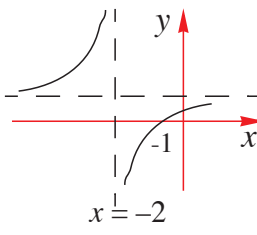
1. (a) $y = 2, x = -1$ (b) $y = 1, x = -\frac{1}{3}$ (c) $y = \frac{1}{2}, x = -\frac{1}{4}$ (d) $y = -1, x = -3$ (e) $y = 3, x = 0$

(f) $y = 5, x = 2$

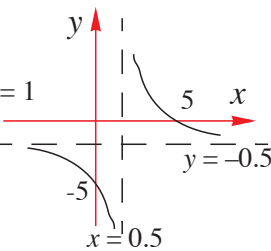
3. (a)



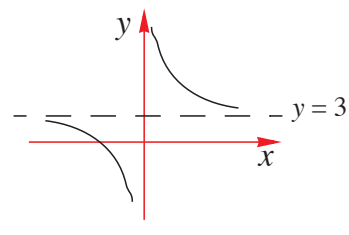
(b)



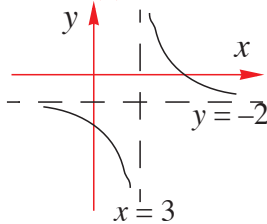
(c)



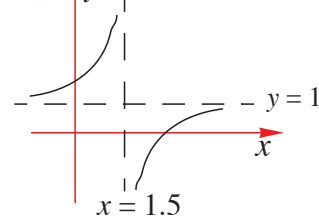
(d)



(e)

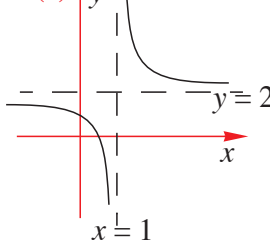


(f)

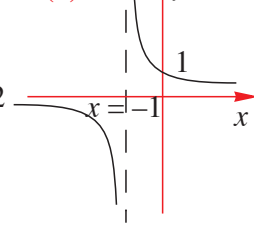


4. $a = 2, c = 4$

5. (a)

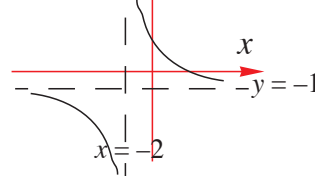


(b)

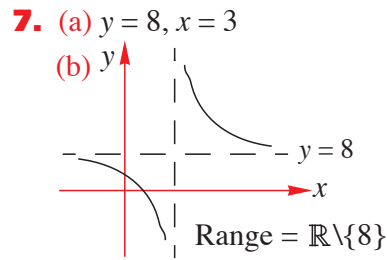
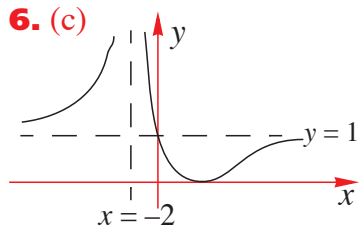


6. (a) i. $(0, 1), (2, 0)$ ii. $y = -1, x = -2$

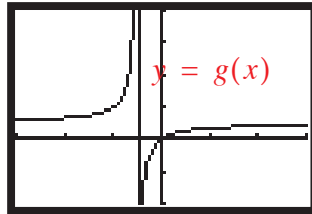
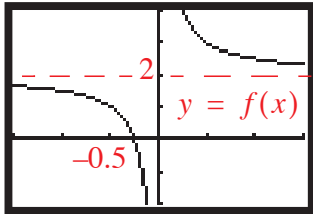
iii. $d = \mathbb{R} \setminus \{-2\}$ iv. $d = \mathbb{R} \setminus \{-2\}$



(b) $f^{-1}: \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R}$, where $f^{-1}(x) = \frac{2(1-x)}{1+x}$



8. dom = $\mathbb{R} \setminus \{0\}$, ran = $\mathbb{R} \setminus \{2\}$ dom = $\mathbb{R} \setminus \{-0.5, 0\}$, ran $\mathbb{R} = \setminus \{0.5\}$



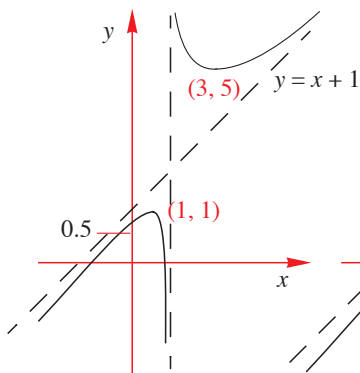
9. asymptotes **(a)** $y = 2x, x = 0$ **(b)** $y = \frac{1}{2}x, x = 0$ **(c)** $y = -x, x = 0$ **(d)** $y = x, x = 0$

10. asymptotes **(a)** $y = x^2, x = 0$ **(b)** $y = x^2, x = 0$ **(c)** $y = x, x = 0$ **(d)** $y = x^3, x = 0$

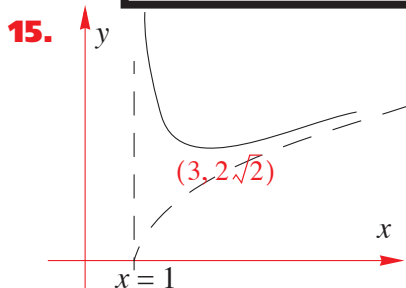
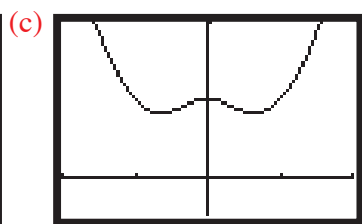
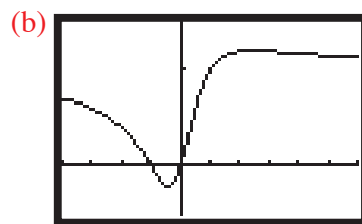
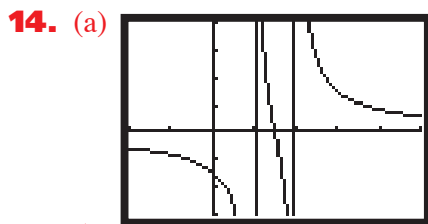
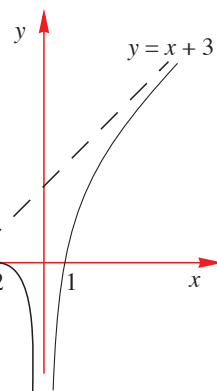
11. asymptotes **(a)** $y = x + 3, x = 0$ **(b)** $y = -x + 2, x = 0$ **(c)** $y = 2x - 2, x = 0$

(d) $y = x + 2, x = 0$ **12. (a)** i. (0, 4), (2, 0) ii. $y = 3 - x, x = 1$

13. i.



ii.



EXERCISE 21.1

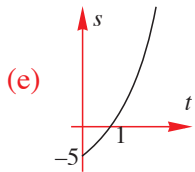
- 1.(a) i. $x < 0$ ii. $x > 4$ iii. $0 \leq x \leq 4$ (b) i. $-1 < x < 2$ ii. $x < -1, 2 < x < 5$ iii. \emptyset
 (c) i. $-1 < x < 1$ ii. $x < -1$ iii. $x \geq 1$ (d) i. $0 < x < 1$ ii. $2 < x < 3$ iii. $x < 0, 1 \leq x < 2$
 (e) i. \emptyset ii. $-2 < x < 4$ iii. \emptyset (f) i. $-4 < x < -1, 2 < x < 5$ ii. $-1 < x < 2, 5 < x < 8$ iii. \emptyset

EXERCISE 21.2

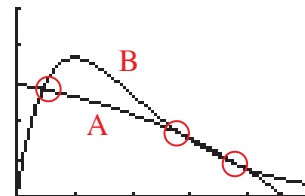
1. 4.4 (4 deer per year, to nearest integer) 2. (a) 200 cm^3 (b) $73.5 \text{ cm}^3 \text{ day}^{-1}$ 3. (a) 75 (b) No
 4. (a) \$207.66 (b) \$40.79 per year (c) \$41.54 per year 5. i. 2.50 ii. 3.33 iii. 2.50
 6. (a) $1230 < x < 48770$ (approx) (b) i. $0 < x < 25000$ ii. $25000 < x < 50000$ 7. 66667 (to nearest integer), 1446992 (to nearest integer) 8. (b) 133.33 (d) 46.67 (e) $0 < x < 5700$
 9. (a) $D'(x) = \frac{-40000(2x+12)}{(x^2+12x+20)^2}$ $5 \leq x \leq 18$ (b) 22.22 (22 items/dollar) 10. (a) $\frac{3000}{(x+32)^2}$
 (b) i. $x \geq 0$ ii. $x \in \emptyset$ 11. (a) i. 0 mm/s ii. $\sim 90.69 \text{ mm/s}$ (b) 0.6 sec 12. (a) 8.53 cm/s (b) (c) never
 13. $-e^{-1} \text{ ms}^{-2}$

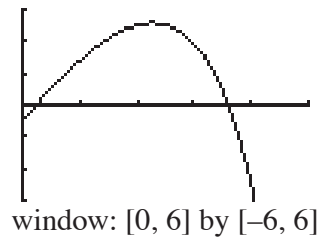
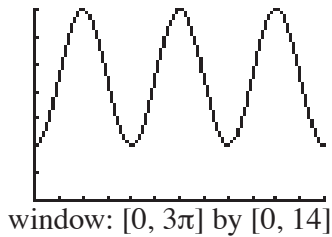
EXERCISE 21.3

1. (a) i. $v = -\frac{1}{(t-1)^2}, t > 1$ ii. $a = \frac{2}{(t-1)^3}, t > 1$ (b) i. $v = 2(e^{2t} - e^{-2t}), t \geq 0$
 ii. $a = 4(e^{2t} + e^{-2t}), t \geq 0$ (c) i. $v = \frac{2}{\sqrt{4-t^2}}, 0 \leq t < 2$ ii. $a = \frac{2t}{(4-t^2)^{3/2}}, 0 \leq t < 2$
 (d) i. $v = \frac{t}{(t+1)\ln 10} + \log_{10}(t+1), t \geq 0$ ii. $a = \frac{1}{\ln 10} \left[\frac{1}{(t+1)^2} + \frac{1}{t+1} \right], t \geq 0$
 (e) i. $v = a - 2bte^{-t^2}, t \geq 0$ ii. $a = 2be^{-t^2}(2t^2 - 1), t \geq 0$
 (f) i. $v = (\ln 2) \times 2^{t+1} - (\ln 3) \times 3^t, t \geq 0$ ii. $v = (\ln 2)^2 \times 2^{t+1} - (\ln 3)^2 \times 3^t, t \geq 0$
 2. (a) 8 ms^{-1} (b) never at rest (c) i. 5m from O in negative direction ii. 4 ms^{-1} (d) 40 m



- (e) 3. (a) 1 ms^{-1} (b) never (c) $t = \frac{1}{3}$ or $t = 1$ (d) 20 ms^{-2}
 4. (a) $v = -6t^2 + 12; a = -12t$ (b) $\sim 1.41 \text{ sec}$ (c) once (d) use graphics calculator
 5. (a) 3 m in positive direction (b) i. 5 m ii. 2 m (c) 5 ms^{-1} (e) oscillation about origin with amplitude 5 m and period $2\pi \text{ sec}$ 7. (a) 100 m, in negative direction (b) 3 times (c) i. 80 ms^{-1}
 ii. -34 ms^{-2} (d) 14.81m 8. (a) max = 5 units, min = -1 unit (b) $\frac{\pi}{2} \text{ sec}$ (c) i. $a = -12 \cos(2t - \pi)$
 ii. $a = -4(x-2)$ 9. (a) 0.318 m above (b) i. $v = 3.75e^{-0.25t} - 3$ ii. $a = -0.9375e^{-0.25t}$
 (c) 0.322 m (d) $a = -0.25(v+3)$ 10. (a) $0 < t < 0.5$ or $t > 1$ (b) $t > 0.5$ (c) $t = 1$ or $1.68 \leq t \leq 5$.
 11. (a) This question is best done using a graphics calculator:
 (b) From the graph the particles pass each other three times.
 (c) 0.45 sec; 2.85 sec; 3.87 sec
 (d) i. $v_A = -0.3e^{0.3t} \text{ ms}^{-1}$ ii. $v_B = 10e^{-t}(1-t) \text{ ms}^{-1}$
 (e) Yes. On two occasions
 12. (a) 2m in positive direction (b) i. 2 sec ii. never (c) 0.026 ms^{-2}
 13. (a) (b) 0.295 14. (a) (b) i. -39.99 ms^{-1} ii. -36.20 ms^{-2}



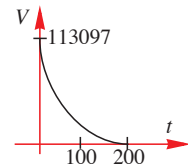


(c) 2.27 sec (d) 5.14 m

EXERCISES 21.4

1. (a) $4\pi r \text{ cm}^2\text{s}^{-1}$ (b) $4\pi \text{ cms}^{-1}$ 2. $6 \text{ cm}^2\text{s}^{-1}$ 3. (a) $\frac{dA}{dt} = -\frac{3}{2}\sqrt{2}x \text{ cm}^2\text{s}^{-1}$ (x = side length)
 (b) $-\frac{3}{2}\sqrt{2} \text{ cms}^{-1}$ 4. (a) $37.5 \text{ cm}^3\text{hr}^{-1}$ (b) $30 \text{ cm}^2\text{hr}^{-1}$ (c) $0.96 \text{ gm}^{-1}\text{cm}^3\text{hr}^{-1}$ 5. $\sim 0.37 \text{ cms}^{-1}$
 6. $-0.24 \text{ cm}^3\text{min}^{-1}$ 7. (a) 0.035 ms^{-1} (b) 0.035 ms^{-1} 8. $8\pi \text{ cm}^3\text{min}^{-1}$ 9. 854 km hr^{-1}
 10. $\frac{53}{6}$ 11. 2 rad s^{-1} 12. i. $V = h^2 + 8h$ ii. $\frac{4}{15} \text{ m min}^{-1}$ iii. $0.56 \text{ m}^2\text{min}^{-1}$
 13. $\frac{3\sqrt{10}}{200} \text{ m min}^{-1}$ 14. $10\sqrt{2} \text{ cm}^3\text{s}^{-1}$ 15. 0.9 ms^{-1} 16. -3.92 ms^{-1}

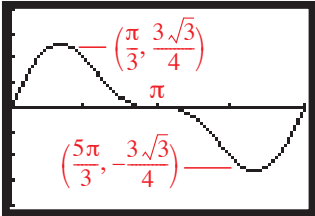
17. (a) $x = 30 - 0.15t$ (b) $[0, 200]$ (c) i. $1531 \text{ cm}^3\text{s}^{-1}$ ii. $15.90 \text{ cm}^2\text{s}^{-1}$ (d)



18. $\sim 1.24 \text{ ms}^{-1}$ 19. $\sim 0.0696 \text{ ms}^{-1}$ 20. (a) $y = \sqrt{119 + 20t - 4t^2}$ (b) $\sim 0.516 \text{ ms}^{-1}$
 21. (a) 0.095 cms^{-1} (b) $0.6747 \text{ cm}^2\text{s}^{-1}$ 22. (a) i. $x = 70t$ ii. $y = 80t$ (b) $130t$ (c) 130 kmh^{-1}
 (d) 14.66 kmh^{-1} 23. -0.77 ms^{-1} 24. 0.40 ms^{-1} 25. 3.2 ms^{-1} 26. 0.075 m min^{-1}
 27. $1.26^\circ \text{ per sec}$ 28. $\frac{5}{2564} \approx 0.002 \text{ rad per sec}$ 29. (a) $9\% \text{ per sec}$ (b) $6\% \text{ per sec}$
 30. 0.064 31. 8211 per year 32. $4\% \text{ per sec}$ 33. $-0.25 \text{ rad per sec}$

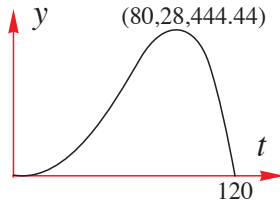
EXERCISES 21.5

1. 22.6 m 2. (a) 1.5 kmh^{-1} (b) $\$19.55 \text{ per km}$ 3. (a) 400 (b) $\$46,400,000$ 4. $\$273.86$
 5. $\$0.40$ 6. 1.97 m 7. 0.45 m^3 8. $5\text{m by } 5\text{m}$ 9. 128
 10. $r = \frac{50}{4 + \pi} \approx 7.00$, dim of rect. $\frac{50}{4 + \pi} \times \frac{50}{4 + \pi}$ i.e., approx $7.00 \text{ m by } 7.00 \text{ m}$ 11. $\theta = \frac{\pi}{6}$

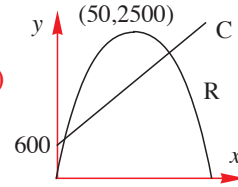
12. (a)  (b) $\frac{3\sqrt{3}}{2}$ units (c) At points of inflections, when $\cos x = -\frac{1}{4}$.

13. 648 m^2 14. (a) 10.5 (b) 5.25 15. 72 16. (a) $y = 100 - 2x$ (b) $A = x(100 - 2x)$, $0 < x < 50$
 (c) $x = 25$, $y = 50$ 17. (a) $\frac{100}{x} - \frac{1}{2}x$, $0 < x < 10\sqrt{2}$ (b) $\frac{2000}{9}\sqrt{6} \approx 544.3 \text{ cm}^3$

18. (a) 400 ml s⁻¹ (b) 40 sec (c)



19. (a)



(b) 8.38, 71.62 (c) $9 \leq x \leq 71$ (d) $80x - x^2 - 600$, \$1000 20. $(\sqrt{\frac{11}{2}}, \frac{7}{2})$ & $(-\sqrt{\frac{11}{2}}, \frac{7}{2})$

21. $5\sqrt{2}$ by $\frac{5}{2}\sqrt{2}$ 22. 4 by $\frac{8}{3}$ 23. $348 - 8\sqrt{170} \sim 243.7$ cm² 24. 2 25. radius = $\sqrt{\frac{10}{3}}$ cm,

height = $2\sqrt{\frac{10}{3}}$ cm 26. $\sqrt[3]{\frac{15}{\pi}}$ 27. 5 cm 28. (a) $h = \frac{24r^2}{r^2 - 144}$ (b) $\frac{8\pi r^4}{r^2 - 144}$ (c) $r = 12\sqrt{2}$,

$h = 48$ 29. $r : h = 1 : 2$ 30. $\sim (0.55, 1.31)$ 31. (b) 2.5 m 32. altitude = $\frac{1}{3}$ height of cone

33. ~ 1.640 m wide and 1.040 m high 34. $\frac{2\sqrt{2}}{\sqrt{3}}\pi$ 35. where $XP : PY = b : a$ 36. 5 km

37. $r : h = 1 : 1$ 38. $\frac{4}{3}$ cm 39. 2 : 1 40. $\frac{10}{\sqrt{3}\pi}$ 41. 0.873 km from P

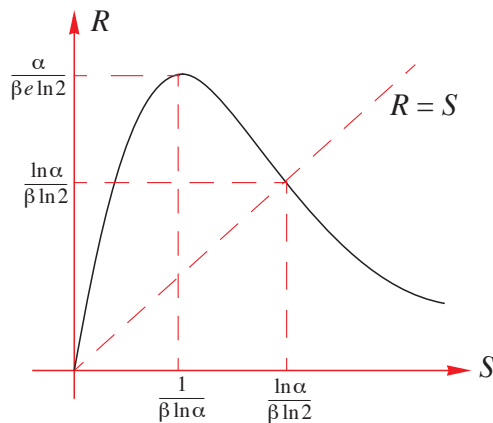
42. (b) $r = 3\sqrt{2}$, $h = 6\sqrt{2}$ 43. (b) when $\theta = \arcsin(\frac{5}{6})$, i.e., approx. 6.030 km from P.

44. (a) $\tan \theta = \frac{xl}{x^2 + k(l+k)}$ (b) $x = \sqrt{k^2 + kl}$

45. (c) if $k < c$, swimmer should row directly to Q. 46. (a) i. $\pi r^2 h + \frac{2}{3}\pi r^3$ ii. $3\pi r^2 + 2\pi r h$

(c) $r : h = 1 : 1$ 47. $(a^{2/3} + b^{2/3})^{3/2}$ 48. (b) 4 km along the beach (c) row directly to destination

49. (a)



(b) First integer greater than $\frac{\ln \alpha}{\beta \ln 2}$

50. $\sqrt{295 \times 145} \approx 207$ m

51. (a) isosceles triangle

(b) isosceles right-angled triangle

52. $r^2(\frac{1}{2} + \frac{1}{4}\sqrt{3})$ sq. units 53. $\frac{4k^2 + 4k - 1}{8k(k+1)}$ sq. units

54. (a) $c = -r$

EXERCISE 22.1

1. (a) $\frac{1}{4}x^4 + c$ (b) $\frac{1}{8}x^8 + c$ (c) $\frac{1}{6}x^6 + c$ (d) $\frac{1}{9}x^9 + c$ (e) $\frac{4}{3}x^3 + c$ (f) $\frac{7}{6}x^6 + c$ (g) $x^9 + c$

(h) $\frac{1}{8}x^4 + c$ 2. (a) $5x + c$ (b) $3x + c$ (c) $10x + c$ (d) $\frac{2}{3}x + c$ (e) $-4x + c$ (f) $-6x + c$

(g) $-\frac{3}{2}x + c$ (h) $-x + c$ 3. (a) $x - \frac{1}{2}x^2 + c$ (b) $2x + \frac{1}{3}x^3 + c$ (c) $\frac{1}{4}x^4 - 9x + c$

- (d) $\frac{2}{5}x + \frac{1}{9}x^3 + c$ (e) $\frac{1}{3}x^{3/2} + \frac{1}{x} + c$ (f) $x^{5/2} + 4x^2 + c$ (g) $\frac{1}{3}x^3 + x^2 + c$ (h) $x^3 - x^2 + c$
- (i) $x - \frac{1}{3}x^3 + c$ **4.** (a) $\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + c$ (b) $\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2 + c$ (c) $\frac{1}{4}(x-3)^4 + c$
- (d) $\frac{2}{5}x^5 + \frac{1}{2}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + c$ (e) $x + \frac{1}{2}x^2 - \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + c$
- (f) $\frac{2}{7}x^{7/2} + \frac{4}{5}x^{5/2} + \frac{2}{3}x^{3/2} - 2x + c$ **5.** (a) $\frac{1}{2}x^2 - 3x + c$ (b) $2u^2 + 5u + \frac{1}{u} + c$
- (c) $-\frac{1}{x} - \frac{2}{x^2} - \frac{4}{3x^3} + c$ (d) $\frac{1}{2}x^2 + 3x + c$ (e) $\frac{1}{2}x^2 - 4x + c$ (f) $\frac{1}{3}t^3 + 2t - \frac{1}{t} + c$
- 6.** (a) $\frac{4}{7}\sqrt[4]{x^7} + 2\sqrt{x} - 5x + c$ (b) $\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{4}{7}x^{7/2} - \frac{4}{5}x^{5/2} + c$ (c) $-\frac{1}{2z^2} + \frac{2}{z} + 2z^2 + z + c$
- (d) $\frac{1}{2}t^4 + t + c$ (e) $\frac{2}{5}\sqrt{t^5} - 2\sqrt{t^3} + c$ (f) $\frac{1}{3}u^3 + 2u^2 + 4u + c$ **8.** (a) $\frac{1}{8}(2x+3)^4 + c$
- (b) $3\sqrt{x^2+4} + c$

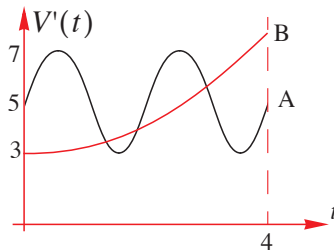
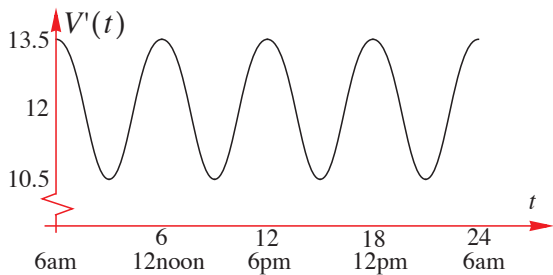
EXERCISE 22.2

- 1.** (a) $x^2 + x + 3$ (b) $2x - \frac{1}{3}x^3 + 1$ (c) $\frac{8}{3}\sqrt{x^3} - \frac{1}{2}x^2 - \frac{40}{3}$ (d) $\frac{1}{2}x^2 + \frac{1}{x} + 2x - \frac{3}{2}$ (e) $(x+2)^3$
- (f) $\frac{3}{4}\sqrt[3]{x^4} + \frac{1}{4}x^4 + x$ (g) $\frac{1}{3}x^3 + 1$ (h) $x^4 - x^3 + 2x + 3$ **2.** $\frac{1}{2}x^2 + \frac{1}{x} + \frac{5}{2}$ **3.** \$3835.03
- 4.** 9.5 **5.** $\frac{251}{3}\pi \text{ cm}^3$ **6.** 292 **7.** $\frac{5}{7}\sqrt{x^3} + \frac{23}{7}$ **8.** (1, -8) **9.** $P(x) = 25 - 5x + \frac{1}{3}x^2$
- 10.** $N = \frac{20000}{201}t^{2.01} + 500, t \geq 0$ **11.** (a) $y = -\frac{2}{5}x^2 + 4x$ (b) $y = \frac{1}{6}x^3 + \frac{5}{4}x^2 + 2x$
- 12.** $y = 2(x^3 + x^2 + x)$ **13.** $f(x) = -\frac{3}{10}x^3 + \frac{49}{10}x - \frac{13}{5}$ **14.** Vol $\sim 43202 \text{ cm}^3$ **15.** 110 cm^2

EXERCISE 22.3

- 1.** (a) $\frac{1}{5}e^{5x} + c$ (b) $\frac{1}{3}e^{3x} + c$ (c) $\frac{1}{2}e^{2x} + c$ (d) $10e^{0.1x} + c$ (e) $-\frac{1}{4}e^{-4x} + c$ (f) $-e^{-4x} + c$
- (g) $-0.2e^{-0.5x} + c$ (h) $-2e^{1-x} + c$ (i) $5e^{x+1} + c$ (j) $e^{2-2x} + c$ (k) $3e^{x/3} + c$ (l) $2\sqrt{e^x} + c$
- 2.** (a) $4\log_e x + c, x > 0$ (b) $-3\log_e x + c, x > 0$ (c) $\frac{2}{5}\log_e x + c, x > 0$ (d) $\log_e(x+1) + c, x > -1$
- (e) $\frac{1}{2}\log_e x + c, x > 0$ (f) $x - 2\log_e x - \frac{1}{x} + c, x > 0$ (g) $\frac{1}{2}x^2 - 2x + \log_e x + c, x > 0$
- (h) $3\ln(x+2) + c$ **3.** (a) $-\frac{1}{3}\cos(3x) + c$ (b) $\frac{1}{2}\sin(2x) + c$ (c) $\frac{1}{5}\tan(5x) + c$ (d) $\cos(x) + c$
- 4.** (a) $-\frac{1}{2}\cos(2x) + \frac{1}{2}x^2 + c$ (b) $2x^3 - \frac{1}{4}\sin(4x) + c$ (c) $\frac{1}{5}e^{5x} + c$ (d) $-\frac{4}{3}e^{-3x} - 2\cos\left(\frac{1}{2}x\right) + c$
- (e) $3\sin\left(\frac{x}{3}\right) + \frac{1}{3}\cos(3x) + c$ (f) $\frac{1}{2}e^{2x} + 4\log_e x - x + c, x > 0$ (g) $\frac{1}{2}e^{2x} + 2e^x + x + c$
- (h) $\frac{5}{4}\cos(4x) + x - \log_e x + c, x > 0$ (i) $\frac{1}{3}\tan(3x) - 2\log_e x + 2e^{x/2} + c, x > 0$

- (j) $\frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + c$ (k) $\frac{1}{2}e^{2x+3} + c$ (l) $-\frac{1}{2}\cos(2x + \pi) + c$ (m) $\sin(x - \pi) + c$
- (n) $-4\cos\left(\frac{1}{4}x + \frac{\pi}{2}\right) + c$ (o) $2\left(\frac{e^x + 2}{\sqrt{e^x}}\right) + c$ **5.** (a) $\frac{1}{16}(4x - 1)^4 + c$ (b) $\frac{1}{21}(3x + 5)^7 + c$
- (c) $-\frac{1}{5}(2 - x)^5 + c$ (d) $\frac{1}{12}(2x + 3)^6 + c$ (e) $-\frac{1}{27}(7 - 3x)^9 + c$ (f) $\frac{1}{5}\left(\frac{1}{2}x - 2\right)^{10} + c$
- (g) $-\frac{1}{25}(5x + 2)^{-5} + c$ (h) $\frac{1}{4}(9 - 4x)^{-1} + c$ (i) $-\frac{1}{2}(x + 3)^{-2} + c$ (j) $\ln(x + 1) + c, x > -1$
- (k) $\ln(2x + 1) + c, x > -\frac{1}{2}$ (l) $-2\ln(3 - 2x) + c, x < \frac{3}{2}$ (m) $3\ln(5 - x) + c, x < 5$
- (n) $-\frac{3}{2}\ln(3 - 6x) + c, x < \frac{1}{2}$ (o) $\frac{5}{3}\ln(3x + 2) + c, x > -\frac{2}{3}$ **6.** (a) $-\frac{1}{2}\cos(2x - 3) - x^2 + c$
- (b) $6\sin\left(2 + \frac{1}{2}x\right) + 5x + c$ (c) $\frac{3}{2}\sin\left(\frac{1}{3}x - 2\right) + \ln(2x + 1) + c$ (d) $10\tan(0.1x - 5) - 2x + c$
- (e) $2\ln(2x + 3) + 2e^{-\frac{1}{2}x+2} + c$ (f) $-\frac{2}{2x+3} - \frac{1}{2}e^{2x-\frac{1}{2}} + c$ (g) $x + \ln(x + 1) - 4\ln(x + 2) + c$
- (h) $2x - 3\ln(x + 2) + \frac{1}{2}\ln(2x + 1) + c$ (i) $-\frac{1}{2x+1} + \ln(2x + 1) + c$
- 7.** (a) $f(x) = \frac{1}{6}\sqrt{(4x + 5)^3}$ (b) $f(x) = 2\ln(4x - 3) + 2$ (c) $f(x) = \frac{1}{2}\sin(2x + 3) + 1$
- (d) $f(x) = 2x + \frac{1}{2}e^{-2x+1} + \frac{1}{2}e$ **8.** 14334 **9.** 13.19ms⁻¹ or 1.19ms⁻¹ **10.** 2.66cm
- 11.** $2e^{x/2} - \frac{1}{2}\sin(2x) - 2$ **12.** (a) $p = \frac{a}{a^2 + b^2}, q = -\frac{b}{a^2 + b^2}$ (b) $\frac{1}{13}e^{2x}(2\sin 3x - 3\cos 3x) + c$
- 13.** (a) 0.25a (b) $a \times \left(\frac{1}{2}\right)^{8/3} \approx 0.1575a$ **14.** (b) 666 gm
- 15.** (a) (b) 73.23% (c) ~25.24 litres **16.** (a) (b) 7000 (c) 1.16 day (d) 2 days



EXERCISES 22.4

- 1.** (a) $\frac{15}{2}$ (b) $\frac{38}{3}$ (c) $\frac{5}{36}$ (d) -8 **2.** (a) $\frac{35}{24}$ (b) $\frac{8}{5}\sqrt{2} - 2$ (c) -2 (d) 0 (e) $\frac{1}{20}$ (f) $-\frac{4}{3}$ (g) $\frac{7}{6}$
- (h) $\frac{5}{6}$ (i) $\frac{20}{3}$ (j) 0 (k) $\frac{20}{3}$ (l) $-\frac{\sqrt{2}}{3}$ **4.** (a) e (b) $2(e^{-2} - e^{-4})$ (c) 0 (d) $2(e - e^{-1})$
- (e) $e^2 + 4 - e^{-2}$ (f) $\frac{1}{2}(e - e^5)$ (g) $2\sqrt{e} - 3$ (h) $\frac{1}{4}(16e^{1/4} - e^4 - 15)$ (i) $\frac{1}{2}(e^{-1} - e^3)$
- 6.** (a) $3\ln 2$ (b) $2\ln 5$ (c) $4 + 4\ln 3$ (d) $\frac{1717}{4}$ (e) $\frac{3}{2}\ln 3$ (f) $2\ln 2$ (g) $\frac{3}{4}$ (h) $4\ln 2 - 2$ (i) $\ln 2$

8. (a) 1 (b) $\frac{3\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) -2 (e) $\frac{\pi^2}{32} - 1$ (f) 0 (g) 0 (h) $\frac{\sqrt{3}}{2} - \frac{1}{2}$ (i) 0 (j) 2 **9.** (a) $\frac{31}{5}$
 (b) $\frac{7\sqrt{7}}{3} - \sqrt{3}$ (c) 0 (d) $\frac{5}{72}$ (e) $3\sqrt[3]{2} - \frac{3}{2}$ (f) $1 - \ln 2$ (g) $\frac{76}{15}$ (h) $\frac{16}{15}$ (i) $\frac{2}{3}(e+1)^{3/2}(1-e^{-3/2})$

10. $\ln\left(\frac{21}{5}\right)$ **11.** $\sin 2x + 2x \cos 2x; 0$ **12.** (a) $2m - n$ (b) $m + a - b$ (c) $-3n$ (d) $m(2a - b)$

(e) na^2 **13.** (a) $e^{0.1x} + 0.1xe^{0.1x}; 10xe^{0.1x} - 100e^{0.1x} + c$ (b) i. 99 accidents

ii. $N = 12t + 10te^{0.1t} - 100e^{0.1t} + 978$ **14.** (a) 1612 subscribers (b) 46220 **15.** (b) ~524 flies

EXERCISES 22.5

1. (a) 4 sq.units. (b) $\frac{32}{3}$ sq.units. (c) 4 sq.units. (d) 36 sq.units. (e) $\frac{1}{6}$ sq.units. **2.** (a) e sq.units.

(b) $\frac{1}{2}(e^4 - 2 - e^2)$ sq.units. (c) $2(e + e^{-1} - 2)$ sq.units. (d) $2(e^2 - 2 - e)$ sq.units.

3. (a) $\ln\left(\frac{5}{4}\right)$ sq.units. (b) $2\ln 5$ sq.units. (c) $3\ln 3$ sq.units. (d) 0.5 sq.units. **4.** (a) 2 sq.units.

(b) $\frac{\pi}{2}$ sq.units. (c) $\frac{3}{8}\pi^2 + \sqrt{2} - 2$ sq.units. (d) $\sqrt{2}$ sq.units. (e) $4\sqrt{3}$ sq.units. **6.** 12 sq.units.

7. $4\left(\sqrt{3} - \frac{1}{3}\right)$ sq.units. **8.** $\ln 2 + 1.5$ sq.units. **9.** 2 sq.units. **10.** $\frac{37}{12}$ sq.units. **11.** (a) 0.5 sq.units.

(b) 1 sq.unit. (c) $2(\sqrt{6} - \sqrt{2})$ sq.units. **12.** $\frac{8}{3}$ **13.** $-2\tan 2x; \frac{1}{4}\ln 2$ sq.units.

14. (a) $\frac{9}{2}$ sq.units. (b) 3 sq.units. **15.** (a) 1 sq.unit. (b) 10 sq.units. **16.** (a) $x \ln x - x + c$

(b) 1 sq.unit. **17.** $\frac{14}{3}$ sq.units **18.** (a) $\frac{7}{6}$ sq.units (b) $\frac{9}{2}$ sq.units **19.** i. $\frac{15}{4}$ sq.units ii. $\frac{45}{4}$ sq.units

20. $\frac{22}{3}$ sq.units. **21.** (b) i. $e^{-1} + e - 2$ sq. units ii. 1 sq. unit iii. $2\ln(2)$ sq. units

22. (b) 3.05 sq. units **23.** (a) $2y = 3ax - a^3$ (b) $\frac{1}{15}a^5$ sq. units **24.** (a) $1 - e^{-1}$ sq. units

(b) e^{-1} sq. units (c) $1 - e^{-e^{-1}-1} - e^{-1} \sim 0.10066$ sq. units **25.** $a = 16$

EXERCISES 22.6

1. (a) $x = t^3 + 3t + 10, t \geq 0$ (b) $x = 4\sin t + 3\cos t - 1, t \geq 0$

(c) $x = t^2 - 4e^{-\frac{1}{2}t} + 2t + 4, t \geq 0$ **2.** (a) $x = t^3 - t^2, t \geq 0$ (b) 100 (c) $100\frac{8}{27}$ m

3. (a) $x = -\frac{2}{3}(4+t)^{3/2} + 2t + 8$ (b) 6.92 m **4.** $\frac{125}{6}$ m **5.** $\frac{125}{49}$ sec; 63.8 m **6.** (a) $\frac{\pi}{6}$ sec

(b) $\frac{\pi}{2} - 1$ m **7.** 80.37 m **8.** (a) $s(t) = \frac{160}{\pi}\left[1 - \cos\left(\frac{\pi}{16}t\right)\right], t \geq 0$ (b) 86.94 m (c) -6.33 m

(d) 116.78 m **9.** (a) $v = 4 + k - \frac{k}{t^2}, t > 0$ (b) $k = 2$ (c) 52.2 m **10.** (b) 0.0893 m

EXERCISES 22.7.1

1. $k = \frac{1}{9}, \frac{1}{27}$ 2. 0.1 3. $k = 8, \frac{5}{16}$ 4. 0.05637 5. ii. 0.0067 6. i. $\frac{1}{\ln 3}$ ii. 0.369 7. ii. 0.135 iii. 1.8% iv. 3 8. i. 0.75 ii. 0.269 iii. 0.1495 iv. 0.575. 10. $\sqrt{2}$

EXERCISES 22.7.2

1. i. Both 0.5 ii. Variance = $\frac{1}{12}$ SD ≈ 0.2887 2. i. Mode = 1, Mean = 0.75, Median ≈ 0.7937
ii. Var = 0.0375 Sd ≈ 0.1936 3. i. All 5gms ii. Var = 0.2, Sd ≈ 0.447 iii. [4.1065, 5.894]
4. i. Mode = 2, Mean = $\frac{4}{3}$, Median ≈ 1.414 ii. Std dev. ≈ 0.4714 iii. Mean. 5. i. 0 ii. ≈ 3.5 sec
iii. 5 sec. iv. 5sec. v. 15sec. 6. All π 7. Mean = 8.5cm Variance = 1.25. 8. (a) $80 \ln\left(\frac{4}{3}\right) \approx 23.01$
(b) $F(t) = 1 - e^{-t/80}, t \geq 0$ (c) Use graphics calculator 9. (b) $a = \frac{3}{7}, b = \frac{5}{2}$ (c) mode = 1.25
10. (a) $k = \frac{1}{2(\sqrt{5}-1)}$ (b) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-1} \approx 0.4078$ (c) 4.62 days (d) 4.75 days
11. (a) $(1-x)^{3/2} - \frac{3}{2}x\sqrt{1-x}$ (b) i. 1.5 ii. $1 - \left(\frac{1}{2}\right)^{2/3} \approx 0.37$; 0 iii. 0.4

EXERCISES 22.8

All values are in cubic units

1. 21π 2. $\pi \ln 5$ 3. $\frac{4}{5}\pi$ 4. $\frac{\pi}{2}(e^{10} - e^2)$ 5. π^2 6. $\frac{\pi}{2}$ 7. $\frac{109}{3}\pi$ 8. $\pi\left(\frac{8}{3} - 2 \ln 3\right)$ 12. $\frac{\pi}{2}(5 - 5 \sin 1)$
13. $\frac{251}{30}\pi$ 14. (a) 40π (b) $\frac{242}{5}\pi$ 15. (a) $\frac{8}{35}\pi$ (b) $\frac{\pi}{4}$ 16. (a) $\frac{9}{2}\pi$ (b) $\frac{88}{5}\sqrt{3}\pi$ 17. $\frac{3\pi}{4}$ 18. $k = 1$
19. $4\pi^2 a^2$ 20. $k = \frac{\pi}{2}$ 21. (b) i. $\frac{\pi a}{2(1+a^2)}$ ii. $\frac{8\pi}{15}\sqrt{\frac{a}{1+a^2}}\left(\frac{3a^2+2}{1+a^2}\right)$ 22. (a) two possible
solutions: solving $a^3 - 6a^2 - 36a + 204 = 0$, $a = 4.95331$; solving $a^3 - 6a^2 - 36a - 28 = 0$,
then $a = -0.95331$. (b) $a = \frac{100}{\pi}$ 23. $\frac{28}{15}\pi$ 24. (a) $\frac{1472}{15}\pi$ (b) 64π (c) $\frac{576}{5}\pi$

EXERCISE 23.1.1

1. (a) $\frac{2}{3}(5x^2+2)^{3/2} + c$ (b) $-\frac{1}{3(x^3+4)} + c$ (c) $\frac{3}{8}(1-2x^2)^4 + c$ (d) $\frac{1}{5}(9+2x^{3/2})^5 + c$
(e) $\frac{9}{4}(x^2+4)^{4/3} + c$ (f) $\frac{-1}{2(x^2+3x+1)^2} + c$ (g) $4\sqrt{x^2+2} + c$ (h) $\frac{1}{12(1-x^4)^3} + c$
(i) $\frac{2}{3}(1+e^{3x})^{3/2} + c$ (j) $\frac{-1}{2(x^2+2x-1)} + c$ (k) $\frac{2}{3}\sqrt{x^3+3x+1} + c$ (l) $\frac{1}{12}(3+4x^2)^{3/2} + c$
(m) $2\sqrt{e^x+2} + c$ (n) $-\frac{1}{4}(1-e^{-2x})^{-2} + c$ (o) $\frac{2}{3}(x^3+1)^5 + c$ (p) $\frac{1}{24}(x^4+8x-3)^6 + c$
(q) $\frac{1}{5}(x^4+5)^{5/2} + c$ (r) $-\sqrt{1-\sin 2x} + c$ (s) $\frac{2}{9}(4+3\sin x)^{3/2} + c$ (t) $-\frac{1}{12(1+3\tan 4x)} + c$

- (u) $\frac{3}{2}(x + \cos x)^{2/3} + c$ (v) $-\frac{1}{2}\cos^4 \frac{x}{2} + c$ (w) $2\sqrt{1+x\sin x} + c$ (x) $\frac{4}{3}(x^{1/2} + 1)^{3/2} + c$
- 2.** (a) $e^{x^2+1} + c$ (b) $6e^{\sqrt{x}} + c$ (c) $\frac{1}{3}e^{\tan 3x} + c$ (d) $-e^{-(ax^2+bx)} + c$ (e) $-6e^{\cos \frac{x}{2}} + c$
- (f) $-4e^{(4+x^{-1})} + c$ (g) $-\frac{1}{2}\cos(2e^x) + c$ (h) $\frac{1}{2(1-e^{2x})} + c$ (i) $-\ln(1+e^{-x}) + c$
- (j) $\frac{5}{2}\ln(1+2e^x) + c$ (k) $-\frac{2}{3a}(4+e^{-ax})^{3/2} + c$ (l) $\frac{(\ln(1+e^{2x}))^2}{4} + c$
- 3.** (a) $-\cos(x^2+1) + c$ (b) $-10\cos\sqrt{x} + c$ (c) $-2\sin\left(2+\frac{1}{x}\right) + c$ (d) $-\frac{2}{3}(\cos x)^{3/2} + c$
- (e) $-\frac{1}{3}\log(\cos 3x) + c$ (f) $\frac{4}{3}\log(1+\tan 3x) + c$ (g) $\frac{-4}{3(\tan(3x)+1)} + c$ (h) $2\sin(\ln x) + c$
- (i) $-\frac{1}{6}(1+\cos 2x)^{3/2} + c$ (j) $\sin(e^x) + c$ (k) $-e^{(-x^3+2)} + c$ (l) $\left[\ln\left(\sin\frac{1}{2}x\right)\right]^2 + c$
- (m) $\sec x + c$ (n) $\frac{1}{4}[\ln(1+2e^x)]^2 + c$ (o) $\tan\left(\frac{1}{3}x^3-3x\right) + c$
- 4.** (a) $\tan^{-1}\left(\frac{x}{2}\right) + c$ (b) $\tan^{-1}\left(\frac{x}{3}\right) + c$ (c) $\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$ (d) $\sin^{-1}\left(\frac{x}{5}\right) + c$ (e) $\sin^{-1}\left(\frac{x}{4}\right) + c$
- (f) $\cos^{-1}\left(\frac{x}{3}\right) + c$ **5.** (a) $3\tan^{-1}x + c$ (b) $5\sin^{-1}x + c$ (c) $\sin^{-1}\left(\frac{x}{2}\right) + c$ (d) $\sin^{-1}\left(\frac{x}{3}\right) + c$
- (e) $\frac{1}{2}\sin^{-1}2x + c$ (f) $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + c$ (g) $\frac{1}{5}\sin^{-1}\left(\frac{5x}{2}\right) + c$ (h) $\tan^{-1}2x$ (i) $\frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right) + c$
- (j) $\frac{1}{12}\tan^{-1}\left(\frac{4x}{3}\right) + c$ (k) $\frac{\sqrt{5}}{15}\tan^{-1}\left(\frac{\sqrt{5}x}{3}\right) + c$ (l) $\frac{1}{\sqrt{5}}\sin^{-1}\left(\sqrt{\frac{5}{3}}x\right) + c$ **6.** (a) $\frac{531377}{9}$
- (b) $-2\sqrt{2} + 2\sqrt{1+e}$ (c) $3\ln 2(2+\sqrt{2})$ (d) $4\tan^{-1}\left(\frac{\pi}{2}\right)$ (e) $\sin e - \sin(e^{-1})$
- (f) $\frac{2}{3}\left[1 - \cos\left(\frac{\pi}{2}\right)^{3/2}\right]$ (g) $\frac{2}{3}$ (h) $e - e^{-1}$ (i) $\ln 2$ (j) $\frac{7\sqrt{7}}{3}$ (k) 0 (l) $\frac{3}{5}$ (m) $\frac{\pi}{4}$ (n) $\frac{\pi}{2} - \tan^{-1}(2)$
- (o) $\frac{1}{3}\tan^{-1}9$ (p) $\frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)$ (q) $\frac{1}{4}\left(\pi - 2\sin^{-1}\left(\frac{2}{3}\right)\right)$ (r) $\frac{1}{3}\tan^{-1}\left(\frac{3}{2}\right)$
- (s) $\frac{1}{6}\left(\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{3}{16}\right)\right)$ (t) $\frac{1}{64}$ (u) $\frac{2}{3} - \frac{\sqrt{2}}{3}$ (v) $\frac{1}{3}$ (w) $\frac{3\pi}{4}$ (x) $-\frac{1}{60}$

EXERCISE 23.1.2

- 1.** (a) $\frac{2}{3}(x^2+1)^{3/2} + c$ (b) $\frac{2}{3}(x^3+1)^{3/2} + c$ (c) $-\frac{1}{3}(4-x^4)^{1.5} + c$ (d) $\ln(x^3+1) + c$
- (e) $-\frac{1}{18(3x^2+9)^3} + c$ (f) $e^{(x^2+4)} + c$ (g) $\ln(z^2+4z-5) + c$ (h) $-\frac{3}{8}(2-t^2)^{4/3} + c$
- (i) $e^{\sin x} + c$ (j) $\ln[e^x+1] + c$ (k) $\frac{1}{5}\sin^5 x + c$ (l) $\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + c$
- 2.** (a) $\frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + c$ (b) $-\frac{2}{3}(1-x)^{3/2} + \frac{4}{5}(1-x)^{5/2} - \frac{2}{7}(1-x)^{7/2} + c$

- (c) $\frac{2}{5}(x-1)^{5/2} + \frac{4}{3}(x-1)^{3/2} + c$ (d) $e^{\tan x} + c$ (e) $-\ln(1-2x^2) + c$ (f) $\frac{1}{1-2x^2} + c$
- (g) $\frac{1}{2}(\ln x)^2 + c$ (h) $-\ln(1+e^{-x}) + c$ (i) $\ln(\ln x) + c$ **3.** (a) 0 (b) $\frac{2\ln 2}{3}$ (c) $\ln \frac{77}{54}$ (d) $\ln 2$
- (e) $\frac{1}{3}\ln 2$ (f) $\frac{1}{4}$ (g) $\frac{76}{15}$ (h) $\frac{16}{15}$ (i) $\frac{2}{3}(1+e)^{3/2}(1-e^{-3/2})$ **4.** (a) $\frac{7\sqrt{7}}{3} - \frac{8}{3}$ (b) $\frac{3}{8}(\cos \pi^2 - 1)$
- (c) $\frac{1042}{5}$ (d) $\ln 4$ (e) 1 (f) $\frac{5}{4}(e^5 - e^{-1})$ (g) 24414 (h) $\sqrt{3} - \sqrt{2}$ (i) $\frac{1}{4}\ln 3$ **5.** (a) $\frac{1}{4}$ (b) $2 - \frac{2}{3}\sqrt{3}$
- (c) $\frac{31}{80}$ (d) $4 - 2\sqrt{2}$ (e) $\ln 2$ (f) $\frac{2}{3}$ **6.** (a) $-\frac{2}{5}\sqrt{3}$ (b) $\frac{2}{5}\sqrt{3}$ (c) $\frac{26}{3}$ (d) $-\frac{4}{3}$ (e) $-\frac{56}{15}\sqrt{2}$ (f) $3 + 2\ln 4$
- 7.** (a) $\tan^{-1}(x+3) + c$ (b) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$ (c) $\sin^{-1}\left(\frac{x-2}{\sqrt{5}}\right) + c$ (d) $3\sin^{-1}\left(\frac{x+1}{3}\right) + c$
- (e) $2\sin^{-1}\left(\frac{2x-3}{\sqrt{29}}\right) + c$ (f) $\frac{1}{2}\sin^{-1}\left(\frac{x^2}{3}\right) + c$ (g) $\frac{1}{2}(\arcsin x)^2 + c$ (h) $-\frac{1}{3}(\arccos x)^3 + c$
- (i) $-\frac{1}{2}(\arcsin x)^{-2} + c$ **8.** (a) $A = 1, B = -2$ **9.** (a) $\tan^{-1} k$ (b) i. $\frac{\pi}{6}$ ii. $\frac{\pi}{4}$ (c) $\frac{\pi}{2}, \pi$
- 10.** $2\sqrt{x} - 2\ln(\sqrt{x} + 1), 2 - 2\ln 2$ **11.** $\frac{3k^2\pi}{8}$ **12.** $\frac{\pi a^2}{4}$

EXERCISE 23.2.1

- 1.** (a) $\sin x - x\cos x + c$ (b) $4\cos \frac{x}{2} + 2x\sin \frac{x}{2} + c$ (c) $2\left(4\sin \frac{x}{2} - 2x\cos \frac{x}{2}\right) + c$
- (d) $-e^{-x}(x+1) + c$ (e) $-5e^{-4x}\left(\frac{x}{4} + \frac{1}{16}\right) + c$ (f) $x\ln x - x + c$ (g) $\frac{x^2}{2}\ln x - \frac{x^2}{4} + c$
- (h) $-\frac{1}{25}(\cos 5x + 5x\sin 5x) + c$ (i) $12\left(x\cos \frac{x}{3} - 3\sin \frac{x}{3}\right) + c$ (j) $\ln \cos x + x\tan x + c$
- (k) $\frac{2}{3}x\sqrt{x}\ln x - \frac{4}{9}x\sqrt{x} + c$ **2.** (a) $\frac{2}{15}(3x-2)(x+1)^{3/2} + c$ (b) $\frac{2}{15}(3x+4)(x-2)^{3/2} + c$
- (c) $\frac{2}{15}(3x+1)(x+2)^{3/2} + c$ **3.** (a) $x\cos^{-1}x - \sqrt{1-x^2} + c$ (b) $x\tan^{-1}x - \frac{1}{2}\ln(x^2+1) + c$
- (c) $x\sin^{-1}x + \sqrt{1-x^2} + c$
- 4.** (a) $\left(\frac{1}{2}x^2 - \frac{1}{4}\right)\cos^{-1}x - \frac{1}{4}x\sqrt{1-x^2} + c$ (b) $\frac{1}{2}(x^2+1)\tan^{-1}x - \frac{x}{2} + c$
- (c) $\frac{1}{4}(2x^2-1)\sin^{-1}x + \frac{1}{4}x\sqrt{1-x^2} + c$ **5.** (a) $\frac{1}{4}$ (b) $\frac{1}{4}(e^2+1)$ (c) $\frac{1}{4}(e^2-4)$ (d) $\frac{1}{4}$
- (e) $\frac{4\pi - \sqrt{2}\pi - 4\sqrt{2}}{32}$ (f) $\frac{1}{2}$ **6.** $\frac{1}{6}\ln 2 + \frac{\pi}{12} - \frac{1}{6}$ **7.** $\frac{1}{2}[\sqrt{2} + \ln(\sqrt{2}+1)]$
- 8.** (a) $\frac{x}{2}\cos(\ln x) + \frac{x}{2}\sin(\ln x) + c$ (b) $-\frac{x}{2}\cos(\ln x) + \frac{x}{2}\sin(\ln x) + c$
- (c) $-\frac{1}{15}(1-x^2)(2+3x^2)\sqrt{1-x^2} + c$

EXERCISE 23.2.2

1. (a) $e^x(x^2 - 2x + 2) + c$ (b) $3\left(\frac{x}{2}\cos 2x + \frac{2x^2 - 1}{4}\sin 2x\right) + c$ (c) $\frac{x^4}{4}\log 2x - \frac{x^4}{16} + c$
 (d) $-\frac{e^x}{5}(2\cos 2x - \sin 2x) + c$ (e) $\frac{2x}{9}\cos 3x + \frac{9x^2 - 2}{27}\sin 3x + c$ (f) $-\frac{e^{-2x}}{4}(\cos 2x - \sin 2x) + c$
 (g) $-8\left(x^3\cos\frac{x}{2} - 6x^2\sin\frac{x}{2} - 24x\cos\frac{x}{2} + 48\sin\frac{x}{2}\right) + c$ (h) $\frac{1}{2}(\ln x)^2 + c$
 (i) $2x - 2x\ln(3x) + x(\ln(3x))^2 + c$ (j) $-\frac{\cos x}{2} - \frac{\cos 3x}{6} + c$
 (k) $\frac{1}{1+a^4}\left(a^3e^{ax}\cos\left(\frac{x}{a}\right) + ae^{ax}\sin\left(\frac{x}{a}\right)\right) + c$ (l) $\left(\frac{2x^3}{7} + \frac{4x^2}{35} - \frac{32x}{105} + \frac{128}{105}\right)\sqrt{x+2} + c$
 (m) $\frac{x^4}{4}\ln ax - \frac{x^4}{16} + c$ (n) $2\sin^{-1}\left(\frac{x}{2}\right) - \frac{x}{2}\sqrt{4-x^2} + c$ (o) $\frac{3}{2}(x\sqrt{x^2-9} + 9\ln(x + \sqrt{x^2-9})) + c$
 (p) $\frac{1}{2}\ln(x^2 + 4) + c$ (q) $x - 2\tan^{-1}\left(\frac{x}{2}\right) + c$ 2. (a) $\frac{\pi^2}{16} - \frac{1}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{1}{2}(e^{2\pi} - e^{\pi/2})$
 (d) $1 - \ln 2 - \frac{1}{2}(\ln 2)^2$ (e) $\frac{a}{a^2 + b^2}\left(e^{\frac{2a\pi}{b}} + e^{\frac{a\pi}{b}}\right)$ (f) $e - 2$

EXERCISE 23.3

1. $\frac{\ln 9}{3}$ 2. (a) 4.66 m (b) 5 m (c) 12.28 m 3. $\frac{\pi}{3}$
 4. (a) (c) i. $\sqrt{2} - 1$ sq. units ii. $2 - \sqrt{2}$ sq. units (d) $\pi\left(\frac{\pi}{4} - \frac{1}{2}\right)$ cubic units
 5. (a) $\frac{4}{5}(1 - e^{-4\pi} + e^{-3\pi} + e^{-\pi})$ (b) $\frac{9\pi}{20}(1 - e^{-8\pi})$ cubic units 6. (a) $\frac{3}{6\sqrt{3} - 2\pi}$ (b) 0.5956

EXERCISE 24.1

5. $m = -3, n = -4$ 7. (a) $k = 2$ or 3 (b) $k = -3$ 8. $a = 2, b = 2$ 9. $a = -2, b = 2$
 10. $ay\frac{dy}{dx} = -x$

EXERCISE 24.2

1. (a) $y = x - \frac{1}{2}\cos 2x + c$ (b) $y = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$ (c) $y = 3\log_e(x^2 + x) + c$
 2. (a) $y = \frac{1}{5}(2t^{5/2} + 3)$ (b) $x = 1 - 2\log_e(\cos 2t)$ (c) $y = \log_{e^{\frac{1}{3}}}(x-1)(x+1) + 4$
 (d) $y = \frac{1}{3} - \frac{1}{(1-x)}$ (e) $x = u + \frac{k}{2t_0^2} - \frac{k}{2(t+t_0)^2}$ (f) $y = \frac{1}{2}\sin(x^2)$ (g) $x = \sin^4 t + 1$

(h) $N = \frac{1}{3}\sqrt{(25+t^2)^3} - \frac{110}{3}$ (i) $T = 9 - \frac{1}{3}\sqrt{(8+\cos 2\theta)^3}$ (j) $P = \frac{10}{9} - \frac{1}{9}e^{-3t} - \frac{1}{3}te^{-3t}$

3. (a) $y = 3\sin(x+c)$ (b) $y = 1 - ke^{-\frac{1}{2}x}$ (c) $y = 2(1 + ke^{2x})^{-1}$

4. (a) $t = 2 - \frac{1}{2N^2}$ (b) $s = 2\tan(2t)$ (c) $p = \frac{1}{2}\log_e(2x)$ (d) $y = \frac{3\tan(3x) + 3\pi}{1 - \pi\tan 3x}$

(e) $y = 4\arcsin(x) + 2$ (f) $y = \frac{98x}{14x+1}$ (g) $\theta = \frac{1}{2}\left(y + \frac{1}{2}\sin 2y - \pi\right)$

(h) $y - 2\arctan\left(\frac{y}{2}\right) = x + 2\pi$ 5. (a) $y = k(x+1)$ (b) $y = ke^{0.5x^2} - 1$

(c) $(y+1)^2 = x^2 + k$ (d) $y = \log_e(c - \cos x)$ (e) $y = \log_e(\sin x - x\cos x + c)$

(f) $e^y(y-1) = \sin x - x\cos x + c$ (g) $y = \sin\left(\frac{1}{2}x^2 + c\right)$

(h) $\frac{1}{2}y\sqrt{1-y^2} + \frac{1}{2}\arcsin y = \log_e x + c$ (i) $y\sqrt{1-y^2} + \arcsin y = x^2 + c$

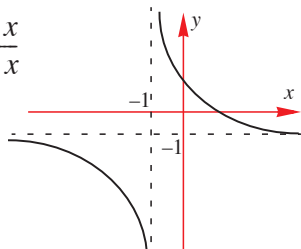
6. (a) $y^2 = \arctan\left(\frac{x}{2}\right) + 1$ (b) $2e^{-y} = e^{-x^2} + 1$ (c) $e^{x-y} = x+1$ (d) $x = 2e^{-0.5(1-t)^2}$

7. (a) $\arctan\left(\frac{y}{2}\right) = \log_e(1+x^2)$ (b) $2\arctan(x) = t^2 - 2$ 8. $x = \frac{1}{2}\left[\frac{3}{(3-y)^2} - 1\right]$

9. $y = 1 - e^{-x}$ 10. $y = \frac{1}{x}e^{(1-x^{-1})}$ 11. $y+1 = \frac{1}{2}\sqrt{x + \frac{1}{2}\sin 2x} + 1$ 16. (a) $\frac{1}{4}\ln\left(\frac{x-2}{x+2}\right) + c$

(b) $y = \frac{2(5+e^{4x})}{5-e^{4x}}$ (c) $y = 2\tan\left(2x - \frac{\pi}{4}\right)$ 17. (a) $\frac{a}{y(a-y)}$ (b) $y = \frac{ae^{akx}}{(a-1) + e^{akx}}$

18. $y = \frac{1-x}{1+x}$



19. $m = -k, n = -2k$

20. (c) $\frac{1}{p-q}\ln\left(\frac{p}{q}\right)$

EXERCISE 24.3

1. 65.61% 2. 31.5% 3. $\frac{dN}{dt} = kN$, 12.68 days. 4. 40.95% 5. 61.05%

7. $y = \frac{1}{2}\log_e(1+x^2)$ 8. 2928 9. $v = 39.2(1 - e^{-t/4})$; 39.2 m/sec 10. 2.5 mins

11. $t \approx 3.31$ 12. 42.5°C 13. (a) i. $x = 80e^{-0.1t}, t \geq 0$ ii. $x = 100 - 20e^{-0.1t}, t \geq 0$

(b) i. $x = 80\left(\frac{50}{50+t}\right)^4, t \geq 0$ ii. $x = 2(50+t) - 20\left(\frac{50}{50+t}\right)^4, t \geq 0$ 15. (b) $\frac{D}{I} \rightarrow \frac{k_1}{k_2}$

17. $I = I_0 e^{\frac{\alpha}{\beta}(e^{-\beta x} - 1)}$, $\alpha, \beta > 0$ 18. $Q = 2Ct - 2RC^2(1 - e^{-t/(RC)})$ 20. $\frac{\sqrt[3]{2}}{\sqrt[3]{2}-1}m$ minutes.

EXERCISE 25.1.1

1. (i) $\begin{bmatrix} 2 & 8 \\ -2 & 16 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 3 \\ 1 & -5 \end{bmatrix}$ (iii) $\begin{bmatrix} 6 & -9 \\ -3 & 15 \end{bmatrix}$ (iv) $\begin{bmatrix} 4 & 5 \\ -3 & 21 \end{bmatrix}$ (v) $\begin{bmatrix} 7 & 6 \\ -5 & 34 \end{bmatrix}$ (vi) $\begin{bmatrix} -2 & -19 \\ 3 & -27 \end{bmatrix}$

2. (i) $\begin{bmatrix} -1 & -1 & -2 \\ 0 & 6 & -9 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & -2 & -4 \\ 0 & 12 & -18 \end{bmatrix}$ (iii) $\begin{bmatrix} -2 & -5 \\ 2 & 2 \\ 3 & 1 \end{bmatrix}$ (iv) $\begin{bmatrix} 2 & 0 \\ 4 & 2 \\ 0 & -2 \end{bmatrix}$ (v) $\begin{bmatrix} 5 & 5 \\ 4 & 1 \\ -3 & -4 \end{bmatrix}$ (vi) $\begin{bmatrix} -6 & -15 \\ 6 & 6 \\ 9 & 3 \end{bmatrix}$

3. (i) $\begin{bmatrix} 3 & 3 & 6 \\ 0 & 3 & 12 \\ 0 & 6 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & -4 & 6 \\ 4 & 0 & 2 \\ 4 & 0 & -8 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & -1 & 5 \\ 2 & 1 & 5 \\ 2 & 2 & -3 \end{bmatrix}$ (iv) $\begin{bmatrix} 2 & 3 & -1 \\ -2 & 1 & 3 \\ -2 & 2 & 5 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & -1 & 12 \\ 4 & 3 & 14 \\ 4 & 6 & -5 \end{bmatrix}$ (vi) $\begin{bmatrix} -3 & -5 & 4 \\ 4 & -1 & -2 \\ 4 & -2 & -9 \end{bmatrix}$

4. (i) 23 (ii) Nuts (iii) Taps (iv) Week 2 Wednesday - = B3 + I3, = B4 + I4 5. 7 by 5 6. $\begin{bmatrix} 3a & 3 \\ 0 & 0 \end{bmatrix}$

7. 3 8. 2 9. (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -\cos 2\theta & 0 \\ 0 & \cos 2\theta \end{bmatrix}$ (c) $\begin{bmatrix} -\cos 2\theta & 0 \\ 0 & \cos 2\theta \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 10. (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -\cos 2\theta & 2\sin\theta \\ -2\cos\theta & \cos 2\theta \end{bmatrix}$ 11. $a = 3, b = 4$ 12. $x = 1$

EXERCISE 25.1.2

1. (i) $\begin{bmatrix} 5 & -4 \\ 2 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 6 & -8 \\ -15 & 26 \end{bmatrix}$ (iii) $\begin{bmatrix} -7 & 17 \\ 3 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} -15 & -10 \\ -10 & -36 \end{bmatrix}$ (v) $\begin{bmatrix} 19 & 89 \\ 6 & 3 \\ 1 & 11 \\ 8 & 2 \end{bmatrix}$ (vi) $\begin{bmatrix} 2.6 & 5.5 \\ -8.55 & 0 \end{bmatrix}$

(vii) $\begin{bmatrix} 12 & -4 & 2 \\ 6 & 0 & -3 \end{bmatrix}$ (viii) $\begin{bmatrix} -9 & 3 & 2 \\ 4 & -2 & -2 \end{bmatrix}$ (ix) $\begin{bmatrix} 7 & 3 & 7 \\ 6 & 2 & 3 \\ 13 & 5 & 29 \\ 3 & 3 & 3 \end{bmatrix}$ (x) $\begin{bmatrix} 1.6 & -0.7 & 3.8 \\ -3.9 & -4.2 & -5.7 \end{bmatrix}$ (xi) $\begin{bmatrix} 11 & -2 \\ 11 & 3 \\ 1 & -7 \end{bmatrix}$

(xii) $\begin{bmatrix} 7 & 5 \\ 21 & -26 \\ 6 & 4 \end{bmatrix}$ (xiii) $\begin{bmatrix} 12 & 6 & 2 \\ 12 & 1 & 4 \\ 21 & 3 & 6 \end{bmatrix}$ (xiv) $\begin{bmatrix} -5 & 6 & 1 \\ 4 & -4 & -12 \\ -1 & 4 & 2 \end{bmatrix}$ (xv) $\begin{bmatrix} -7 & 2 & 13 \\ 8 & -8 & -17 \\ -4 & -11 & 4 \end{bmatrix}$ (xvi) $\begin{bmatrix} -\frac{25}{2} & -\frac{50}{3} & 4 \\ -\frac{79}{4} & -\frac{83}{6} & \frac{27}{4} \\ -2 & \frac{11}{6} & \frac{3}{4} \end{bmatrix}$

(xvii) $\begin{bmatrix} x + 3x^2 & -x^2 + 1 \\ 2x + 5x^2 & -x^2 + 2 \\ -x^2 + 5x & -2x \end{bmatrix}$ (xviii) $\begin{bmatrix} a + 2x^2 - a^2 & 2a + 4x - 2x^2 + 2a \\ a - 2ax & -2a & 3a + 2ax \\ 0 & 2a & -ax - x \end{bmatrix}$

2. (i) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ if n is odd and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if n is even.

3. (i) $\begin{bmatrix} 0 & 3 & 4 & 2 \\ 1 & 0 & 0 & 6 \\ 1 & 2 & 0 & 2 \\ 4 & 3 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 120 \\ 105 \\ 110 \\ 100 \end{bmatrix}$ (ii) $\begin{bmatrix} 955 \\ 720 \\ 530 \\ 795 \end{bmatrix}$ (iii) Loading costs for each depot. **4.** $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} n = 4.$

5. (a) 2×1 (b) $-$ (c) 3×2 (d) $-$ (e) $-$ (f) 1×3 **6.** (a) No (b) Yes (c) $AB = BA$ **7.** $\begin{bmatrix} n \\ \frac{12-5n}{4} \end{bmatrix}, n \in \mathbb{R}$

8. $a = -0.4, b = 2$ **9.** B **10.** (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, S^3 = S$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, S^{2n+1} = S$ (c) $0, \pm 2\pi, \pm 4\pi, \dots$

11. $x = 0 = y, w = z$ **12.** 2, 6 **13.** $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & p^2 \\ p & 1 & -(2p+1) \end{bmatrix}, a = 2p, b = p-1, c = p$

15. $x = 3, y = -2, a = -9, b = -4$

17. (a) $A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}, A^4 = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ (b) $A^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$

EXERCISE 25.2

1. (i) -4 (ii) 9 (iii) 12 (iv) 2 (v) 15 (vi) $\frac{4}{9}$ (vii) 0.5 (viii) 4 (ix) 1 (x) $x^2 - 2$ (xi) 0 (xii) 1

2. (i) $\begin{bmatrix} 0 & 1 \\ 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & -1 \\ 2 & \frac{3}{2} \end{bmatrix}$ (iii) $\begin{bmatrix} -\frac{3}{14} & \frac{1}{7} \\ -\frac{2}{7} & -\frac{1}{7} \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}$ (v) $\begin{bmatrix} 0 & 1 \\ -1 & -4 \end{bmatrix}$ (vi) no inverse

(vii) $\begin{bmatrix} -\frac{1}{5} & \frac{1}{5} \\ \frac{4}{15} & -\frac{1}{15} \end{bmatrix}$ (viii) $\begin{bmatrix} 1 & -\frac{1}{2} \\ 1 & -\frac{3}{4} \end{bmatrix}$ (ix) $\begin{bmatrix} -4 & -1 \\ 1 & -1 \end{bmatrix}$ (x) $\begin{bmatrix} 1 & 3 \\ 1 & 0 \end{bmatrix}$ (xi) $\begin{bmatrix} 2 & 3 \\ 2 & -3 \end{bmatrix}$ (xii) $\begin{bmatrix} 3 & 3 \\ 1 & -2 \end{bmatrix}$ (xiii) $\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$

(xiv) $\begin{bmatrix} \frac{3}{x} & -\frac{1}{x} \\ -2 & 1 \end{bmatrix}$ (xv) $\begin{bmatrix} -1 & 1 \\ \frac{3}{x} & -\frac{2}{x} \end{bmatrix}$ (xvi) $\frac{x-1}{-x^3+x^2+2x-1} \begin{bmatrix} \frac{1}{x-1} & 1-x \\ -x-1 & x \end{bmatrix}$ **3.** $-2, 2$ **4.** -24 **5.** $-\frac{1}{2}$

6. (a) -29 (b) 80 (c) 0 (d) -11 **7.** (a) $\frac{1}{29} \begin{bmatrix} 5 & -7 & 13 \\ -7 & 4 & 5 \\ 4 & 6 & -7 \end{bmatrix}$ (b) $\frac{1}{80} \begin{bmatrix} 9 & 10 & 7 \\ -1 & -10 & 17 \\ 14 & -20 & 2 \end{bmatrix}$ (c) does not exist.

(d) $\frac{1}{11} \begin{bmatrix} 2 & 3 & -1 \\ 5 & 2 & 3 \\ 4 & 6 & 9 \end{bmatrix}$ **8.** $-5 \frac{1}{5} \begin{bmatrix} 1 & -2 & 4 \\ -1 & 2 & 1 \\ 2 & 1 & -2 \end{bmatrix}$ **9.** $p = 2 \frac{1}{35} \begin{bmatrix} 5 & 4 & 7 \\ 10 & -6 & 7 \\ 5 & 11 & -7 \end{bmatrix}$ **11.** $-\frac{23}{4} \frac{1}{23} \begin{bmatrix} -15 & 6 & -8 \\ 26 & 8 & 20 \\ -4 & -3 & 4 \end{bmatrix}$

12. $x = 1$

$$13. -2 \begin{bmatrix} 11 & -9 & -10 \\ -1 & 1 & 1 \\ 8 & -\frac{13}{2} & -\frac{15}{2} \end{bmatrix} \quad 15. (a) AB = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}, A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$(b) AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \quad (c) AB = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}, A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

16. (a) 0,1,3 (b) -2,0 18. (d) $\alpha = 14, \beta = 3$ (e) 1 (or 0, but we will assume that A is not singular). 21. (a) -2 (b) $k = -1$

EXERCISE 25.3

1. (i) (1,-1) (ii) (4,-3) (iii) (-3,-3) (iv) (-2,-1) (v) (2,5) (vi) (1,0) (vii) (0,3) (viii) (-2,4)

(ix) (0.5,2) (x) (2,0.6) (xi) (-1,-3) (xii) (-2,2) (xiii) $(3, \frac{1}{3})$ (xiv) (5,-2) (xv) (5,4)

2. (i) $a = -2, b = -4$ (ii) $a = -2, b \neq -4$ 3. ± 2 4. (a) (1,-3,2) (b) (5,-3,-1) (c) (-0.6,-4.2,-1.4)

5. (78,29,-47) 6. $(\frac{4}{11}, \frac{9}{11}, \frac{7}{11})$ 7. (a) $m \neq -1, 2$ (b) $m = -1$ (c) $m = 2$ 8. (a) $p \neq 1, 2$

(b) $p = 1, 2, k \neq 6$ (c) $p = 1, 2, k = 6$ 9. (a) (4,0) (b) \emptyset (c) (2,-3) (d) (2, 3, -2)

(e) $(4 + 2\lambda, 3 - \lambda, \lambda)$ (f) (1, 2, -2) (g) \emptyset (h) $(2\lambda - 1, \lambda, \lambda)$ (i) \emptyset

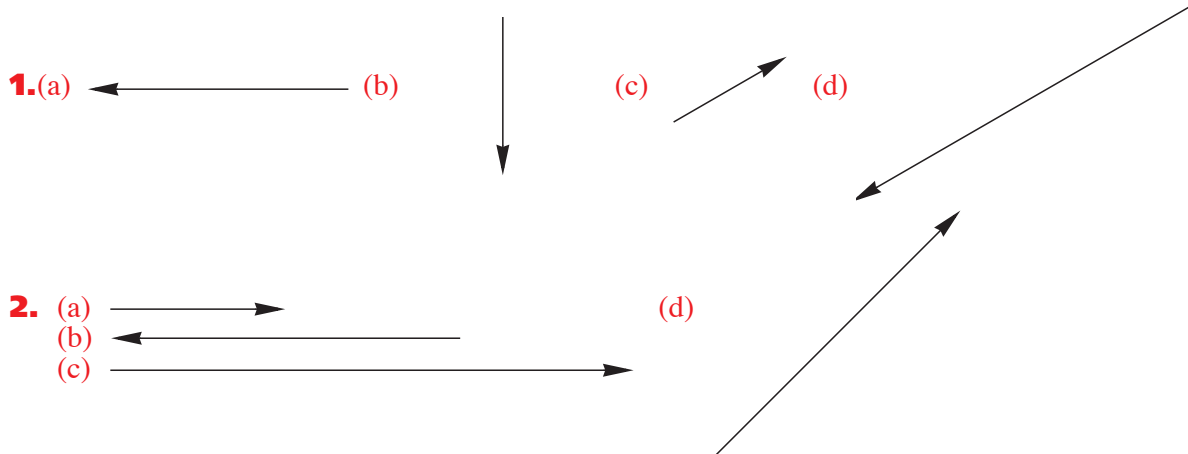
10. $y = x^2 + 6x - 14$ 11. $y = -(\frac{\lambda}{2} + 6)x^2 + (\frac{\lambda}{2} + 13)x + \lambda$ 12. (a) i. $(-\frac{7\lambda}{11}, -\frac{\lambda}{11}, \lambda)$

ii. $(\frac{7-7\lambda}{11}, \frac{1-\lambda}{11}, \lambda)$ (b) (0,0,0) 13. (a) 1,-2 (b) i. 1,-2 ii. neither 1 nor -2 iii. \emptyset

EXERCISE 26.1

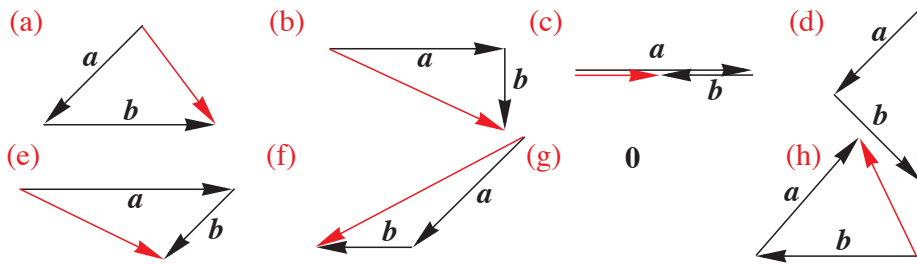
1. vector 2. scalar 3. scalar 4. vector 5. vector 6. vector 7. scalar 8. scalar

EXERCISE 26.2



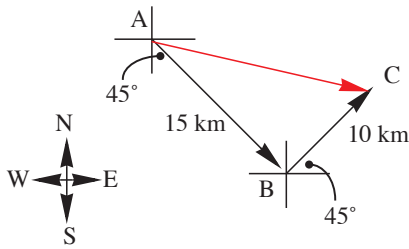
3. (a) {a,b,e,g,u}; {d,f} (b) {d,f}; {a,c}; {b,e} (c) {a,g}; {c,g} (d) {d,f}; {b,e} (e) {d,f}; {b,e}; {a,c,g}

4.



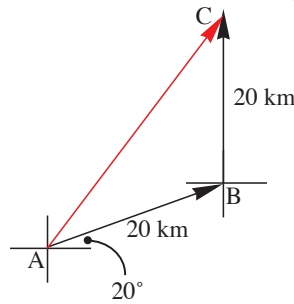
5. (a) AC (b) AB (c) AD (d) BA (e) 0 6. (a) Y (b) N (c) Y (d) Y (e) N

7. (a) i.



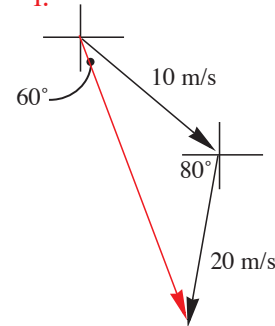
ii. $\sqrt{325}$

(b) i.



ii. $20\sqrt{2(1 - \cos 110^\circ)}$

(c) i.



ii. $10\sqrt{5 - 4\cos 110^\circ}$

8. 72.11 N, E $33^\circ 41'$ N 9. 2719 N along river 10. (b) i. 200 kph N ii. 213.6 kph, N $7^\circ 37'$ W

11. i. 200 ii. 369.32

EXERCISE 12.3

1. (a) $c - a$ (b) $b - c$ (c) $\frac{1}{2}(b + a)$ 2. (a) $b - a$ (b) $b - 2a$ (c) $2b - 3a$ (d) $\frac{1}{2}(b + 2a)$

3. (a) 0 (b) PS (c) AY (d) 6OC 4. (a) $\frac{1}{2}(b + a)$ (b) $\frac{1}{3}(2b + a)$ (c) $\frac{1}{4}(a + b + 2c)$

7. (a) $c - b$ (b) $c + a$ (c) $a + c - 2b$ 8. (a) $2\sqrt{21}$ (b) $2\sqrt{26}$ 15. $m = \frac{13}{23}, n = \frac{50}{23}$ 16. $m = \frac{4}{3}$

EXERCISE 12.4

1. (i) $4i + 28j - 4k$ (ii) $12i + 21j + 15k$ (iii) $-2i + 7j - 7k$ (iv) $-6i - 12k$

2. (i) $3i - 4j + 2k$ (ii) $-8i + 24j + 13k$ (iii) $18i - 32j + k$ (iv) $-15i + 36j + 12k$

3. (i) $\begin{pmatrix} 11 \\ 0 \\ 8 \end{pmatrix}$ (ii) $\begin{pmatrix} -27 \\ 1 \\ -22 \end{pmatrix}$ (iii) $\begin{pmatrix} -3 \\ -6 \\ 12 \end{pmatrix}$ (iv) $\begin{pmatrix} 16 \\ -1 \\ 14 \end{pmatrix}$ 4. $\begin{pmatrix} -5 \\ 3 \end{pmatrix}$ 5. $\begin{pmatrix} -2 \\ 3 \end{pmatrix}, (-2, 3)$

6. (i) $8i - 4j - 28k$ (ii) $-19i - 7j - 16k$ (iii) $-17i + j + 22k$ (iv) $40i + 4j - 20k$

7. (i) $\begin{pmatrix} 20 \\ 1 \\ 25 \end{pmatrix}$ (ii) $\begin{pmatrix} 12 \\ 2 \\ 16 \end{pmatrix}$ (iii) $\begin{pmatrix} -4 \\ -38 \\ -32 \end{pmatrix}$ (iv) $\begin{pmatrix} -20 \\ -22 \\ -40 \end{pmatrix}$ 8. $A = -4, B = -7$

9. i. $(2, -5)$ ii. $(-4, 3)$ iii. $(-6, -5)$ 10. Depends on basis used. Here we used: East as i , North j and vertically up k (b) $D = 600i - 800j + 60k, A = -1200i - 300j + 60k$ (c) $1800i - 500j$

EXERCISE 12.5

1. (i) $\sqrt{10}$ (ii) $5\sqrt{2}$ (iii) $\sqrt{30}$ (iv) 3 (v) $\sqrt{53}$ (vi) $\sqrt{41}$ (vii) $\sqrt{14}$ (viii) $\sqrt{17}$

2. (i) $\frac{1}{\sqrt{2}}(i+j)$ (ii) $\frac{1}{\sqrt{41}}(4i+5j)$ (iii) $\frac{1}{\sqrt{5}}(-i-2j)$ (iv) $\frac{1}{\sqrt{46}}(i+6j-3k)$ (v) $\frac{1}{\sqrt{5}}(i+2k)$

(vi) $\frac{1}{\sqrt{17}}(2i-2j-3k)$ (vii) $\frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ (viii) $\frac{1}{3\sqrt{3}}\begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix}$ **3. i.** Depends on the basis: $-3i+4j+k$

or $-4i-3j+k$ ii. $\sqrt{26}$ **4.** (a) $\sqrt{3}(i-j+k)$ (b) $\frac{1}{4}(3i-j+\sqrt{2}k)$ **5.** $\pm\sqrt{11}$ **6.** $\sqrt{13}$

EXERCISE 12.6

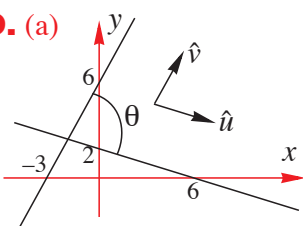
1. (a) 4 (b) -11.49 (c) 25 **2.** (i) 12 (ii) 27 (iii) -8 (iv) -49 (v) 4 (vi) -21 (vii) 6 (viii) -4 (ix) -10 **3.** (i) 79° (ii) 108° (iii) 55° (iv) 50° (v) 74° (vi) 172° (vii) 80° (viii) 58° **4.** (i) -8 (ii) 0.5

5. (a) -6 (b) 2 (c) not possible (d) 5 (e) not possible (f) 0 **6.** (a) $4-2\sqrt{3}$ (b) $2\sqrt{3}-4$

(c) $14-2\sqrt{3}$ (d) not possible **7.** 1 **8.** 105.2° **9.** $x = -\frac{16}{7}, y = -\frac{44}{7}$ **10.** $\pm\frac{1}{\sqrt{11}}(-i+j+3k)$

12. (a) $\lambda(-16i-10j+k)$ (b) e.g. $i+j+\frac{3}{7}k$ **14.** $a \perp b-c$ if $b \neq c$ or $b = c$ **15.** (a) $\left(\frac{3}{5}, \frac{4}{5}\right)$

(b) $\left(\frac{\sqrt{2}}{2}, \frac{1}{2}, -\frac{1}{2}\right)$ **16.** (a) $\left(-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ (b) $131.8^\circ, 48.2^\circ, 70.5^\circ$ **18.** (a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$

19. (a)  (b) $\hat{u} = \frac{1}{\sqrt{10}}(3i-j), \hat{v} = \frac{1}{\sqrt{5}}(i+2j)$ **20.** $\frac{1}{2}(-i+j+\sqrt{2}k)$
(c) 81.87°

25. (a) Use i as a 1 km eastward vector and j as a 1 km northwards vector (b) $\overrightarrow{WD} = 4i+8j$, $\overrightarrow{WS} = 13i+j$ and $\overrightarrow{DS} = 9i-7j$ (c) $\frac{1}{\sqrt{80}}(4i+8j)$ (d) $\frac{d}{\sqrt{80}}(4i+8j)$ (e) $3i+6j$

EXERCISE 12.7.1

1. (a) i. $r = i+2j$ ii. $r = -5i+11j$ iii. $r = 5i-4j$ (b) line joins (1,2) & (5,-4)

2. (a) $r = 2i+5j+\lambda(3i-4j)$ (b) $r = -3i+4j+\lambda(-i+5j)$ (c) $r = j+\lambda(7i+8j)$

(d) $r = i-6j+\lambda(2i+3j)$ (e) $r = \begin{pmatrix} -1 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} -2 \\ 10 \end{pmatrix}$ or $r = -i-j+\lambda(-2i+10j)$

(f) $r = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 5 \\ 1 \end{pmatrix}$ or $r = i+2j+\lambda(5i+j)$

3. (a) $r = 2i+3j+\lambda(2i+5j)$ (b) $r = i+5j+\lambda(-3i-4j)$ (c) $r = 4i-3j+\lambda(-5i+j)$

4. (a) $r = 9i+5j+\lambda(i-3j)$ (b) $r = 6i-6j+t(-4i-2j)$ (c) $r = -i+3j+\lambda(-4i+8j)$

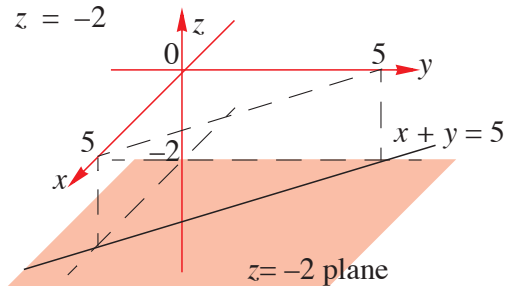
(d) $r = i+2j+\mu\left(\frac{1}{2}i-\frac{1}{3}j\right)$ **5.** (a) $x = -8+2\mu$ (b) $x = 7-3\mu$ (c) $x = 5+2.5\mu$
 $y = 10+\mu$ $y = 4-2\mu$ $y = 3+0.5\mu$

- (d) $x = 0.5 - 0.1t$
 $y = 0.4 + 0.2t$ **6.** (a) $\frac{x-1}{3} = y-3$ (b) $\frac{x-2}{-7} = \frac{y-4}{-5}$ (c) $x+2 = \frac{y+4}{8}$
- (d) $x-0.5 = \frac{y-0.2}{-11}$ (e) $x = 7$ (f) $y = 6$ **7.** (a) $r = 2j + t(3i + j)$ (b) $r = 5i + t(i + j)$
- (c) $r = -6i + t(2i + j)$ **8.** (a) $6i + 13j$ (b) $-\frac{16}{3}i - \frac{28}{3}j$ **9.** $r = 2i + 7j + t(4i + 3j)$
- 11.** (a) $(4, -2), (-1, 1), (9, -5)$ (b) -2 (d) $r = 4i - 2j + \lambda(-5i + 3j)$ (e) i. $M \parallel L$ ii. $M = L$
- 12.** $4x + 3y = 11$ **13.** (a) $\frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}}$ (b) $\frac{4}{5}, \frac{3}{5}$ **14.** (b) ii. & iii. **15.** $(-83, -215)$
- 16.** $r = \frac{k}{7}(19i + 20j)$ **17.** (a) $(\frac{92}{11}, \frac{31}{11})$ (b) \emptyset (c) Lines are coincident, all points are common.

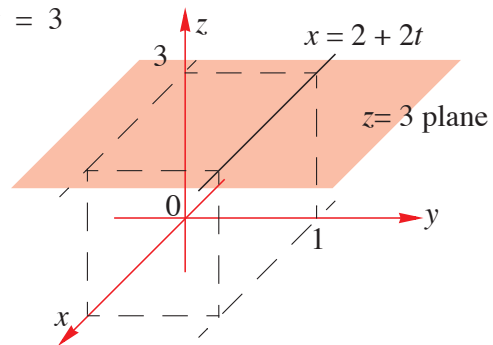
EXERCISE 12.7.2

- 1.** (a) $r = 2i + j + 3k + t(i - 2j + 3k)$ (b) $r = 2i - 3j - k + t(-2i + k)$
- 2.** (a) $r = 2i + 5k + t(i + 4j + 3k)$ (b) $r = 3i - 4j + 7k + t(4i + 9j - 5k)$
- (c) $r = 4i + 4j + 4k + t(7i + 7k)$ **3.** (a) $\frac{x}{3} = \frac{y-2}{4} = \frac{z-3}{5}$ (b) $\frac{x+2}{5} = \frac{z+1}{-2}, y = 3$
- (c) $x = y = z$ **4.** $x = 5 - 7t$
 $y = 2 + 2t$ $r = \begin{pmatrix} 5 \\ 2 \\ 6 \end{pmatrix} + t \begin{pmatrix} -7 \\ 2 \\ -4 \end{pmatrix}$ $\frac{x-5}{-7} = \frac{y-2}{2} = \frac{z-6}{-4}$ **5.** $(\frac{13}{5}, \frac{23}{5}, 0)$
 $z = 6 - 4t$
- $x = 2 + 3t$ $x = 1 + 1.5t$ $x = 3 - t$ $x = 1 + 2t$
- 6.** (a) $y = 5 + t$ (b) $y = t$ (c) $y = 2 - 3t$ (d) $y = 3 + 2t$
 $z = 4 + 0.5t$ $z = 4 - 2t$ $z = 4 + 2t$ $z = 2 + 0.5t$
- 7.** (a) $\frac{x-4}{3} = \frac{y-1}{-4} = \frac{z+2}{-2}$ (b) $x = 2, y = \frac{z-1}{-3}$ **9.** (a) $\frac{x+1}{2} = y-3 = \frac{z-5}{-1}$
- (b) $\frac{x-2}{2} = \frac{z-1}{-2}, y = 1$ **10.** (a) $(1, -1, 0)$ (b) $a = 15, b = -11$

11. (a) $x = 1 + t$
 $y = 4 - t$
 $z = -2$



(b) $x = 2 + 2t$
 $y = 1$
 $z = 3$



$r = \begin{pmatrix} 1 \\ 0.5 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1.5 \\ 1 \end{pmatrix}$. Line passes through $(1, 0.5, 2)$ and is parallel to the vector $2i - \frac{3}{2}j + k$

- 13.** (a) 54.74° (b) 82.25° (c) 57.69° **14.** (a) $(4, 10.5, 15)$ (b) Does not intersect.

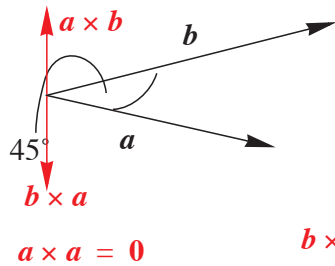
15. (a) L: $x = \frac{y-2}{2} = \frac{z}{5}$, M: $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ (b) \emptyset (c) 84.92° (d) i. $(0, 2, 0)$ ii. $(0, \frac{1}{2}, 0)$

18. $\frac{x}{4} = \frac{y}{9} = \frac{z}{3}$ 19. $k = -\frac{7}{2}$ 20. 64° 21. 3 or -2 22. $12i + 6j - 7k$ (or any multiple thereof) 23. Not parallel. Do not intersect. Lines are skew.

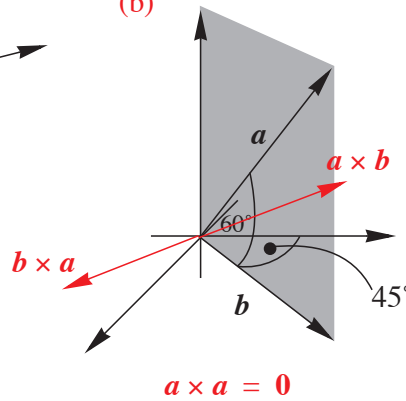
EXERCISE 27.1.1

1. (a) 5 (b) $4\sqrt{3}$ (c) 0 (d) 6 (e) 0

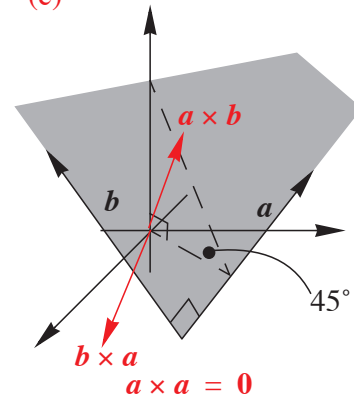
2. (a)



(b)



(c)



3. i. $2\sqrt{91}$ ii. 16 4. $56^\circ 27'$ 5. $3\sqrt{5}$ 6. i. $2\sqrt{3}$ ii. $5\sqrt{3}$

EXERCISE 27.1.2

1. i. $-12i + 4k$ ii. $10i - 2j - 2k$ iii. $18i - 9j$ iv. $10i + 2j - 2k$ v. $-6i + 9j + 8k$

vi. $20i - 13j - 4k$ 2. $-10i + 6j - 2k$ 5. (a) 0 6. $\frac{-6i + 2j - 2k}{\sqrt{11}}$ 7. (a) λk

(b) $\lambda(9i - 3j + 9k)$ 8. (a) 90° (b) 79.1° 12. They must be parallel.

EXERCISE 27.1.3

1. (a) $\sqrt{54}$ (b) $\sqrt{234}$ 2. (a) $\frac{1}{2}(-3i - 13j + 29k)$ (b) $\frac{1}{2}\sqrt{1019}$ (c) 67.84° 3. $\frac{1}{2}\sqrt{2331}$ 4. $12\sqrt{2}$

5. $43^\circ 36'$ 6. $\sqrt{293}$ sq. units 7. $\frac{1}{2}\sqrt{35}$ 9. (a) $\mathbf{OA} = \cos\alpha i + \sin\alpha j$, $\mathbf{OB} = \cos\beta i + \sin\beta j$

12. 66 cubic units 13. (b) $k = 0.5$

EXERCISE 27.2.1

1. (a) $r = i + k + \lambda(3i + 2j + k) + \mu(-2i - j + k)$

(b) $r = -i + 2j + k + \lambda(i - j + 2k) + \mu(-i - j + k)$

(c) $r = 4i + j + 5k + \lambda(2i + 2j - k) + \mu(2i - j + 3k)$

(d) $r = 2i - 3j - k + \lambda(-3i + j - 2k) + \mu(i - 2j + \frac{1}{2}k)$

2. (a) $3x - 5y + z = 4$ (b) $x - 3y - 2z = -9$ (c) $5x - 8y - 6z = -18$ (d) $7x + y - 10z = 21$

3. i. (a) $r = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix}$ (b) $x + 3y - 2z = 3$ ii. (a) $r = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 5 \\ -11 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$

(b) $13x + 3y - z = 31$ **4.** (a) i. $\mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ ii. $\mathbf{r} = \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

(b) i. $x + 7y - 5z = -27$ ii. $x + 7y - 5z = 0$ (c) $-i - 7j + 5k$ (d) Coefficients are just negative of those in (b)

EXERCISE 27.2.2

1. (a) $2x - y + 5z = 7$ (b) $-4x + 6y - 8z = 34$ (c) $-x + 3y - 2z = 0$ (d) $5x + 2y + z = 0$

2. (c) & (d) **3.** (a) $-3x - y + 2z = 3$ (b) $y = 2$ (c) $2x + 2y - z = -3$ **4.** (a) 29.5° (b) 70°

(c) 90° (d) 11° **5.** (a) 83° (b) 50° (c) 49° **6.** (a) $2x + y + 2z = 12$ (b) $8x + 17y - z = 65$

7. $x - 2y + 3z = -2$ **8.** $3x - 2y + 5z = -2$ **9.** $a = \frac{24}{13}$, $b = \frac{18}{13}$

10. (a) $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + t(2\mathbf{i} + 5\mathbf{j} + 5\mathbf{k})$ (b) $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ (c) 49.8°

EXERCISE 27.2.3

1. (a) $2x + 7y - 6z = 33$; $\mathbf{r} \cdot \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix} = 33$ (b) $x - 2y = 0$; $\mathbf{r} \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = 0$

2. $3x - y - z = 2$ **5.** (a) 3 (b) $\frac{5}{3}$ (c) $\sqrt{11}$ (d) $\frac{20}{\sqrt{21}}$ **6.** $3x + 4y - 5z = -4$

7. $2x + 3y - 3z = 5$ **9.** $x + 5y - 6z = -19$

EXERCISE 27.3.1

1. (a) $(7, 5, -3)$ **2.** Lines that intersect are (b) and (c); $(7, -4, 10)$; 46.7° **3.** $(5, -2, -3)$ **4.** $(4, 0, 6)$

EXERCISE 27.3.2

1. (a) i. $(7, 4, 2)$ ii. 36.3° (b) i. $(5, 2, -5)$ i. 10.1° (c) i. $(6, -5, -7)$ ii. 4.4° (d) i. $(3, -1, 1)$ ii. 29.1°

2. (a) $\left(\frac{3}{2}, \frac{5}{2}, 2\right)$ (b) $(0, 4, 1)$ **3.** (a) Plane is parallel to the z -axis slicing the x - y plane on the line $x + y = 6$. (b) $x = 4$ forms a plane. $y = 2z$ is in this plane parallel to the y - z plane. $(4, 2, 1)$ **4.** 13

EXERCISE 27.3.3

1. (a) $x = -2y + 9 = -2z - 3$ or $\frac{x+3}{-2} = y - 6 = z$; $22^\circ 12'$ (b) $\frac{21 - 10x}{-9} = y = \frac{7 - 10z}{7}$

or $\frac{7x - 29}{-11} = \frac{7y + 9}{-6} = z$; $70^\circ 48'$ (c) planes parallel

(d) $x = 4, z = -2 - 2y$; $65^\circ 54'$ **3.** $73^\circ 42'$

EXERCISE 27.3.4

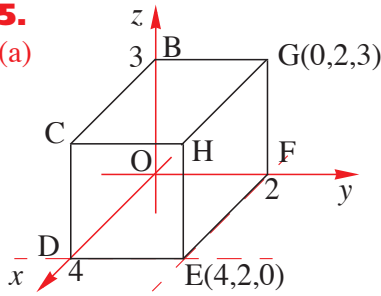
1. E.g. the faces of a triangular prism. **2.** (a) $4 - 2x = \frac{2y - 4}{-5} = z$ or $\frac{x}{1} = \frac{y + 8}{5} = \frac{z - 4}{-2}$

3. (a) $5 - 4x = y = \frac{8 - 4z}{7}$ or $\frac{x}{1} = \frac{y - 5}{-4} = \frac{z + 6}{7}$ **4.** (a) no solution (b) unique solution

(5,1,4) (c) unique solution (5,1,-3) (d) intersect on plane $\frac{5x+19}{-8} = \frac{5y-13}{1} = z$

5.

(a)



$x = 4t \quad x = 4s$

(b) $y = 2t \quad y = 2s \quad (2,1,1.5)$ (c) $3x + 6y - 4z = 12$

$z = 3t \quad z = 3 - 3s$

(d) $\left(\frac{8}{3}, \frac{4}{3}, 1\right)$ $58^\circ 52'$ (e) 59.2°

6. None of these planes are parallel but the lines of intersection of pairs of planes are skew.

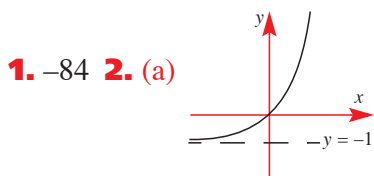
7. $k = 2; \mathbf{r} = \begin{pmatrix} 0 \\ 3.5 \\ 1.5 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2.5 \\ -0.5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$ **8.** (a) $-2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ (b) $\mathbf{r} = t \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$

(c) i. 5 iii. not 5

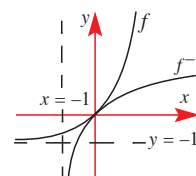
9. (b) $(a - b + c, a + b - c, -a + b + c)$ (c) $\left(\frac{1}{a} - \frac{1}{b} + \frac{1}{c}, \frac{1}{a} + \frac{1}{b} - \frac{1}{c}, -\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

10. (a) 2, 3 (b) 3 (c) For $k = 2, x - 1 = 4 - y = z$

REVISION EXERCISES - SET A

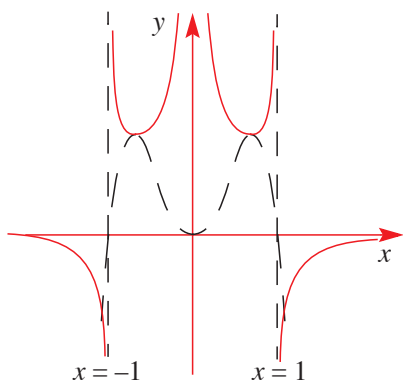


(b) i. $]-1, \infty[$ ii. $f^{-1}(x) = \ln(x+1)$ (c)

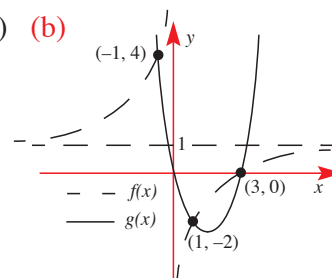


3. 840 **4.** (a) i. 0 ii. 2 (b) $-2 \leq x \leq 2$ (c) $x \geq 0$ **5.** (a) Absolute maximum at $(\pm \frac{1}{\sqrt{2}}, 1)$; local min

at $(0, 0)$; x-intercept at $(\pm 1, 0)$ (b) Local min at $(\pm \frac{1}{\sqrt{2}}, 1)$; asymptotes at $x = \pm 1, y = 0$.

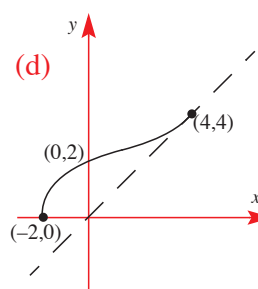
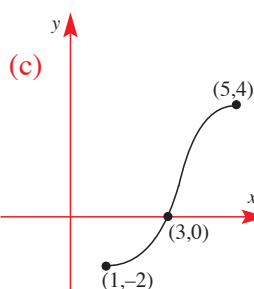
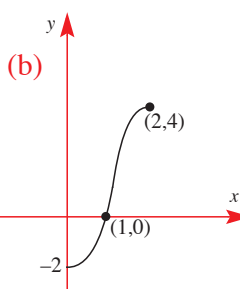
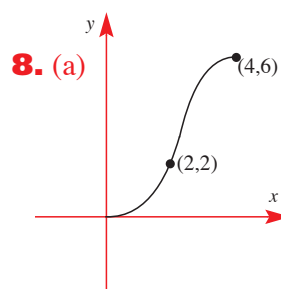


6. (a) $(1, -2), (-1, 4)$ and $(3, 0)$ (b)



7. (a) 2 (b) $S = [0, \infty[$, range = $[1, \infty[$

(c) $f^{-1} : [1, \infty[\mapsto \mathbb{R}, f^{-1}(x) = (\ln x)^2$



9. (a) i. 512 ii. 2 (b) i. $3x^2h + 3xh^2 + h^3$ ii. $3x^2 + 3xh + h^2$

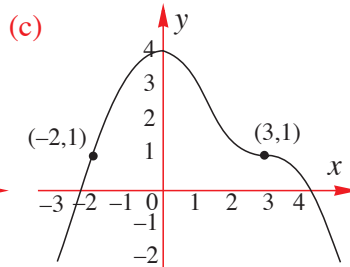
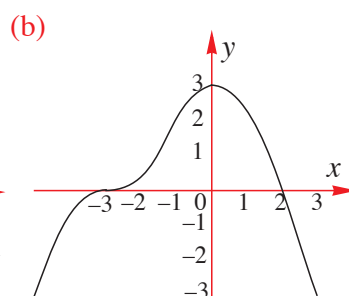
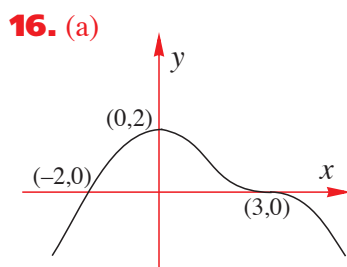
10. (a) i. -1 or 6 ii. $\frac{3}{e-1}$ (b) i. $\mathbb{R} \setminus \{3\}$ ii. 0.2 iii. 0

11. (a) i. 2 or 6 ii. $\frac{1}{3}(e^2 - 4)$ (b) i. $0 < x < 1$ ii. \mathbb{R} iii. $\log_e 4 \approx 0.72$ iv. $\frac{e^{0.8}}{1 + e^{0.8}} \approx 0.69$

12. (a) $g(f(x)) = -\frac{2x}{1-x}, x \in \mathbb{R} \setminus \{\pm 1\}$ (b) $P = (2, 4)$

13. (a) i. $x = \frac{\ln 6}{\ln 3}$ ii. $\frac{15}{7}$ (b) $1 + \sqrt{3}$ (c) i. $f(g(x)) = \sqrt{\frac{1}{x^2} - 1}, g(f(x)) = \frac{1}{x-1}$ ii. $[-1, 1] \setminus \{0\}$

14. 0.5 **15.** (a) $1, 1 + 2\sqrt{3}, 1 - 2\sqrt{3}$ (b) Cubic through $(1 - 2\sqrt{3}, 0), (1, 0), (1 + 2\sqrt{3}, 0)$ with local maximum at $(-1, 16)$ and local minimum at $(3, -16)$ (c) i. $k < 16$ ii. $k = \pm 16$ iii. $-16 < k < 16$



17. (a) $k = 0$ or 16 (b) $(2x - 1)(3x + 2)(x + 3)$ (c) $0 < x < 3$ **18.** (a) $0 < x < 5$ (b) 70 (c) $-2, -\frac{1}{2}, 1$

19. (a) i. $P(x) = (x + 3)(x - 2)(2x - 1)$ ii. $\left\{x \mid -3 < x < \frac{1}{2}\right\} \cup \{x \mid x > 2\}$ (b) i. 9 ii. -4

20. (a) ± 3 (b) $2x^3 - 2x^2 - x + 1 - \frac{4}{2x - 1}$ (c) $-2 < x < 0$ or $x > 2$ **21.** (a) $\frac{1}{3} < x < 5$ (b) $y = -2x$

(c) $\frac{x - y}{x + y}$ **22.** $a = 1, b = 6$ **23.** (b) $x = -\frac{4}{9}, y = \frac{1}{9}$ **24.** (b) ii. $p^5 = 3 + 5p, p^{-5} = 5p - 8$

(c) $\frac{1}{2} < x < \frac{2}{3}$ or $x > \frac{3}{4}$ **25.** (a) $\frac{2}{9}$ (b) 59136 (c) $P(x) = (x + 3)(x + 1 + \sqrt{2})(x + 1 - \sqrt{2})$

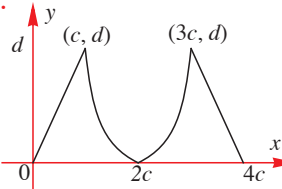
(d) $a = 2$ & $b = 1$ or $a = -1$ & $b = -8$ **26.** (b) $(x - 1)^2(3 - x)$ (c) $\{x \mid x < 1\} \cup \{x \mid 1 < x < 3\}$

27. (a) $-5 < k < 3$ (b) $p = 1, q = 0$ (c) i. $\{x \mid -6 < x < -3\} \cup \{x \mid -1 < x < 4\}$ ii. $\{x \mid x < 1\}$

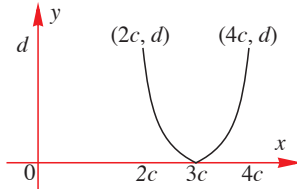
28. $a = -\frac{3}{5}, b = -\frac{648}{25}, n = 10$ **29.** (a) ii. $\{\pm 1\}$ (b) i. $y = \sqrt{6(x - 3)}$ ii. $x = 9, y = 6$

30. $1792x^5$ **31.** (a) $\frac{5}{2}, -\frac{3}{2}$ (b) $\frac{3}{2}, -\frac{1}{2}$ (c) $\frac{17}{2}$ **32.** (a) \mathbb{R} (b) $]-\infty, 4]$ (c) $]-\infty, 4[$ **33.** (b) $\frac{5}{8}$ sq. units

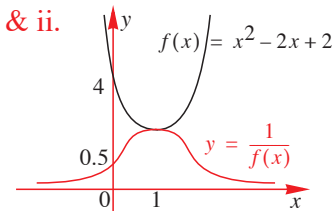
34. i.



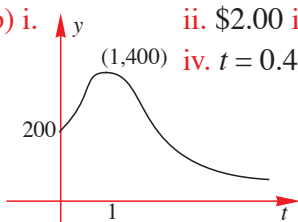
ii.



35. (a) i. & ii.



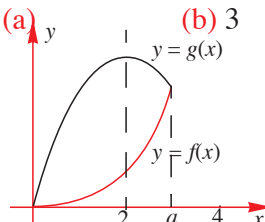
(b) i.



ii. \$2.00 iii. \$4

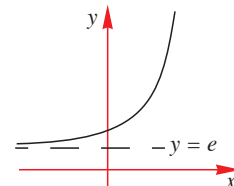
iv. $t = 0.42, 1.57$

36. (a)

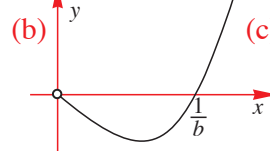


(b) 3

37. $g^{-1}(x) = e^{2x} + e, x \in \mathbb{R}$



38. (a) i. $]0, \infty[$ ii. $\left] \left(-\frac{a}{eb}\right), \infty\right[$

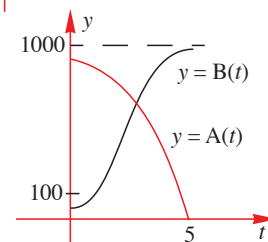


(c) $(1, a \log_e b)$ (d) $x = b^{\frac{1}{x} - 1}$

39. (a) $a = -36, b = 900$

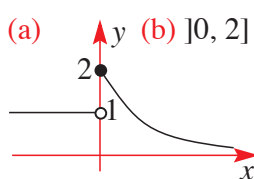
(b) (c) 20 (d) 1000 (e) $t > 1.95$ (f)

t	1	2	3.5	5
$B(t)$	131.04	527.02	957.23	997.78



40. (a) 150 cm (b) 138 cm (c) 94 hrs (d) $[0, 94]$ (e) $h^{-1}(x) = \frac{12.5 - \sqrt{t}}{0.13}$ (f) Use g.c (g) 17.3 hrs

41. $x = -8, y = 11, z = -6$ **42.** (a)

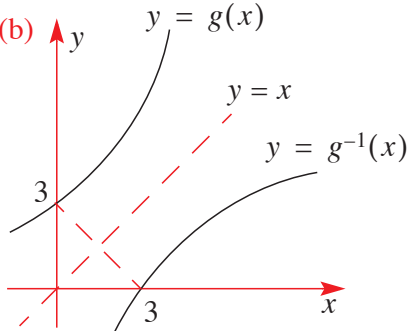


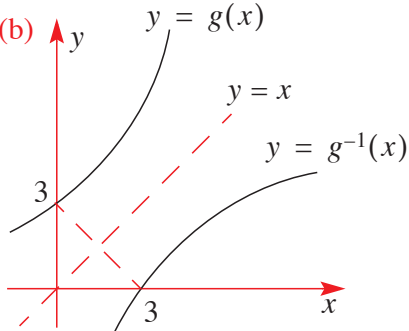
43. 78

44. (a) 0 (b) $-\sqrt{2}$

(c) $r_f \not\subseteq d_g$

i.e., does not exist

45. $-\frac{63}{8}x^5$ 46. (a) $g^{-1}(x) = -1 + \sqrt{x-2}, x \geq 2$ (b)  (c) no



47. ii. $h(x) = 4 - x, x \geq 0$, range = $]-\infty, 4]$ 48. i. use g.c ii. $f^{-1}(x) = -\log_e(1-x), x < 1$

iii. use g.c 49. -10 50. $x = 7\lambda, y = \lambda, z = -11\lambda, \lambda \in \mathbb{R}$

51. (a) $r_g \subseteq d_f \Rightarrow f \circ g$ exists; $r_f \not\subseteq d_g \Rightarrow g \circ f$ doesn't exist (b) $x < -2$ or $x > 2$

52. $x = \lambda, y = 2 - \lambda, z = 3 + 2\lambda, \lambda \in \mathbb{R}$ 53. (a) $x = \lambda, y = \frac{8-3\lambda}{5}, z = \frac{6-\lambda}{5}, \lambda \in \mathbb{R}$

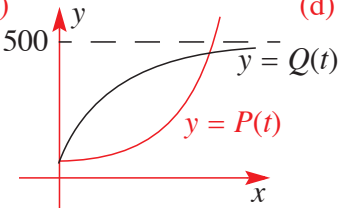
(c) $S =]-3, 2[$ 54. (a) $f^{-1}(x) = (2-x)^2, x < 2$ (b) $r_g \not\subseteq d_{f^{-1}} \Rightarrow f^{-1} \circ g$ does not exist;

$r_{f^{-1}} \subseteq d_g \Rightarrow g \circ f^{-1}$ exists (c) $F(x) = x - 2, x \leq 2$

55. (a) $t = 2$ or 3 (b) $t = 3$ (c) $x = 1 + \lambda, y = 4 - \lambda, z = \lambda, \lambda \in \mathbb{R}$

56. (a) i. 50 ii. $50e \approx 135.9$

(c)  (d) i. 50 ii. 334.5 (f) Increasing at a decreasing rate



(g) ~ 460 wasps

(h) ii. $t = 0$ and $t = 10 \log_e 9$

REVISION EXERCISES - SET B

1. (a) 189 (b) 99 (c) -96 (d) 36 2. (b) -65 3. (b) 23.9 km (c) 26.4° (d) 15.3 km (e) 107.5

4. (a) i. A: \$49000; B: \$52400; C: \$19200 ii. A: \$502400; B: \$506100; C: \$379400 (b) 4.6%

(c) i. 14 months ii. C never reaches its target 5. (a) $\frac{1}{2}\sqrt{26}$ (b) $-\frac{1}{2}(5-i)$ (c) $-\frac{1}{2}$ 6. (a) $r = 0.5$

(b) 62.5 cm 7. (b) $26^\circ 34'$ or 135° (c) $-2\sqrt{3}$ 8. (a) i. $\frac{1}{2}$ ii. $\frac{1}{3}$ iii. $\frac{1}{(k+1)(k-1)!}$

9. (b) $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$ 11. (a) 28 (b) i. $\frac{13}{36}$ ii. $\frac{17}{2}$ 12. (a) $3 + 4i$ or $-3 - 4i$ (b) $\text{cis}(2\theta)$

13. (a) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ (b) $0, \frac{\pi}{2}, \pi, 2\pi$ (c) i. $R = 1, \alpha = \frac{\pi}{6}$ ii. $\left\{\frac{\pi}{2}, \frac{\pi}{6}\right\}$ 15. (a) max value is $\frac{17}{2}$

for $x = \frac{\pi}{2} + 2k\pi$ or $x = \frac{3\pi}{2} + 2k\pi$, where k is an integer; min value is $\frac{17}{5}$ for $x = k\pi$, where k

is an integer (b) $\frac{\pi}{3}, \frac{5\pi}{3}$ 16. (a) $u_n = 74 - 6n$ (b) $n = \frac{1}{6}(74 - p)$ (c) $\frac{1}{12}(74 - p)(68 + p), 420$

17. (a) $P(x) = (x^2 - 2x + 2)(x + 3)$ (b) $P(x) = (x - 1 - i)(x - 1 + i)(x - 3)$

18. $\frac{24(4\sqrt{3}-3)}{39}$ 19. i. $\frac{\pi}{2}$ ii. $\frac{1}{2}$ 22. (a) $60^\circ, 109^\circ 28', 250^\circ 32', 300^\circ$ (b) i. $2\operatorname{cosec}\theta$ ii. $\frac{\pi}{3}, \frac{2\pi}{3}$

23. (a) $z = 2 - 2i, z^2 = -8i$ (b) $-0.96 + 0.72i$ (c) $z = -2 \pm \sqrt{3}, z = \frac{1}{2} \pm \frac{1}{2}\sqrt{3}i$ 26. $k = \frac{\pi}{4}$

27. (a) ~ 342 (b) 20 terms (c) $0 < x < 2$ (d) $\{1, 3, 8, 18, \dots\}$ (e) $u_n = 23 - 3n$ (f) \$4131.45

28. (a) $-\frac{1}{2}$ (b) 4 29. (a) 120° (b) $14\sqrt{3}$ cm² 30. (a) i. $0.3\sqrt{3}$ m ii. $0.2\sqrt{3}$ m (b) ~ 1.15 m

(c) $73^\circ 13'$ 31. (a) $\frac{\pi}{3}, \frac{4\pi}{3}$ (b) $\left\{x \mid \frac{\pi}{3} < x < \frac{4\pi}{3}\right\}$ 32. (a) 8 cm (b) $28^\circ 4'$ 33. (a) $\frac{5\pi}{6}$ (b) i. $64 + 0i$

ii. $4(\sqrt{3} - i)$ iii. $0 - 2i$ iv. $\frac{1}{64}(\sqrt{3} + i)$ v. $-128 - 128\sqrt{3}i$ 34. (a) i. $\frac{1}{32}$ ii. $\frac{1}{64}(-1 + \sqrt{3}i)$

(b) $\pm\sqrt{2}i$ or $1 \pm i$ 35. 3 36. (a) $\left\{\frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}\right\}$ (b) $\left(\frac{\pi}{3}, 1\right)$ 37. (a) \$77156.10

(b) $u_1 = -\sqrt{3}, u_3 = -3\sqrt{3}$ 38. (a) $f(x) = 3\cos(2x)$ (b) $\left\{\frac{7\pi}{6}\right\}$ (c) 3 39. (b) i. BP = 660 m,

PQ = 688 m 40. 216° 41. (b) 906 m 42. (a) $38^\circ 40'$ (b) 0.08004 m² (c) \$493.71 43. (a) 5

(b) i. $\frac{4}{27}$ ii. $-\frac{11\pi}{12}$ (c) $a = -8$ 44. (a) $\tan\alpha = -\frac{1+\sqrt{5}}{2}$ (b) range = $[3, 3.5]$ (c) i. 3 ii. 2

(e) domain = $[-\sqrt{3}, \sqrt{3}]$

45. (a) i. $W(4) = 19.38, P(4) = 14.82$ ii. $W(20) = 10.95, P(20) = 27.02$

iii. $W(35) = 13.45, P(35) = 23.25$ (b) Amp = 5, period = 50 weeks

(c)  (d) \$27.07 (e) during 7th & 46th week

46. (a) \$49000, \$47900, \$46690 (b) \$34062.58

(c) 18.8 yrs (d) \sim \$248564

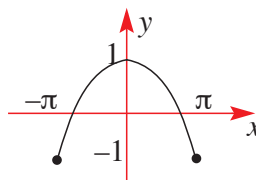
47. (a) ii. 26 cards (b) 26, 40, 57, 77

(c) $a = 3, b = -1$ (d) 155 cards

(e) $t_n = \frac{n}{2}(3n + 1)$

48. (a) ~ 2.77 m (b) i. 3.0 m ii. 2.0 m (c) 4.15 pm


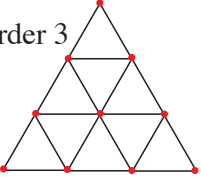
(d) use g.c. (e) $2\frac{1}{6} < t < 6\frac{1}{3}$

49. 1.262 ha 50. $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ 51. (a) $x = -\frac{2\pi}{3}, \frac{2\pi}{3}$ (b)  (c) $-\frac{2\pi}{3} < x < \frac{2\pi}{3}$

52. 1623 m 53. (a) 19.5°C (b) $D(t) = -1 + 2\cos\left(\frac{\pi}{12}t\right)$ (d) use g.c (e) 8 am to midnight

54. 1939 m 55. (a) ii. $N_0 = 2000, \alpha = 10$ (b) 2000, 2200, 2420, 2662, 2988.2 (c) 52 hrs (d) 176995

56. (a) $(4 - \pi)$ cm² (b) $\frac{(4 - \pi)}{2}$ cm² (c) ii. $r = \frac{1}{2}$ iii. $A_n = (4 - \pi) \times \left(\frac{1}{2}\right)^{n-1}, n = 1, 2, \dots$

- (d) i. $\frac{31}{16}(4 - \pi) \text{ cm}^2$ ii. $2(4 - \pi) \text{ cm}^2$ (e) geometric **57.** (a) Order 1  Order 3 
 (b) 4 (c) Order 3: 9, Order 4: 16 & Order 5: 25 (d) n^2
 (e) Order 1: 3, Order 2: 6, Order 3: 10, Order 4: 15
 (f) $a = 1, b = 3, c = 2$ (h) Ratio tends to 0.5. This means that for large grids there are approximately twice as many small triangles as there are nodes.

REVISION EXERCISES - SET C

- 1.** (a) $\frac{1}{4}$ (b) $\frac{3}{8}$ (c) 0.3169 **2.** 0.0228 **3.** (a) 0.12 (b) 0.6087 **4.** 0.0527

- 5.** (a) $P(X = x) = \frac{\binom{5}{x} \binom{10}{5-x}}{\binom{15}{5}}, x = 0, 1, 2, 3, 4, 5$ (b) $E(X) = \frac{5}{3}, \text{var}(X) = \frac{50}{63}$ **6.** (a) 0.89 (b)

- $\frac{21}{40}$ (c) $\frac{40}{89}$ **7.** (a) 0.46 (b) $\frac{9}{23}$ **8.** (a) 3326400 (b) i. $\frac{2}{11}$ ii. $\frac{2}{77}$ **9.** (a) 0.9772 (b) 0.3413

- 10.** (a) 2 (b) 0.3233 **11.** (a) 0.936 (b) 5 **12.** (a) $X \sim \text{Hg}(n = 4, D = 4, N = 8)$ (b) $E(X) = 2,$
 $\text{var}(X) = \frac{4}{7}$ **13.** (a) 792 (b) 35 **14.** (a) 151200 (b) 0.1512 **15.** 0.2852 **16.** (a) 0.0067 (b)

- Poisson distribution with parameter $\lambda = \frac{2}{3}$ (c) 0.5134 **17.** $\frac{128}{850} \approx 0.1506$ **18.** (a) 0.10 (b) 0.40

- (c) $(x, P(X = x))$ values are: (0, 0.40), (1, 0.50), (2, 0.10) (d) $E(X) = 0.70, \text{var}(X) = 0.41$

- 19.** (a) 0.8664 (b) 0.7210 (c) 0.9034 (d) $9.8855 < Y < 10.2145$ (e) 79.3350

- 20.** (a) 315 (b) 17280 **21.** $\frac{193}{512}$ **22.** (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ **23.** (a) $P(X = x) = \frac{1}{6} \times \left(\frac{5}{6}\right)^x, x = 0, 1, \dots$

- (i.e., Geometric) (b) i. 0.0670 ii. 0.4019 iii. $\frac{1}{6}$ **24.** (a) $\frac{13}{44}$ (b) $\frac{9}{44}$

- 25.** (b) $(x, P(X = x))$ values are: $\left(1, \frac{9}{25}\right), \left(3, \frac{7}{25}\right), \left(5, \frac{5}{25}\right), \left(10, \frac{3}{25}\right), \left(20, \frac{1}{25}\right)$

- (c) $E(X) = \frac{105}{25} \approx 4.2, \text{var}(X) = \frac{11400}{625} \approx 18.24$ (d) 0.00064 **26.** (a) 0.3085 (b) 0.0091 (c) 0.1587

- 27.** 100 **28.** (a) $\frac{1}{2}$ (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ **29.** (a) 0.5940 (b) ~ 34 days **30.** (b) $(x, P(X = x))$ values

- are: (1,0.4), (2,0.3), (3,0.2), (4,0.1) (c) i. 2 ii. 5 iii. 3 **31.** (a) 2, 1.35 (b) $\frac{2}{18}$ (c) Binomial

- (d) $\frac{4}{3}$ (e) i. 0.38 ii. 0.390 **32.** (a) i. $\frac{3}{7}$ ii. 0.3456 (b) 4.87% (c) i. 0.8186 ii. 0.1585

- 33.** (a) $(x, P(X = x))$ values are: $\left(0, \frac{3}{16}\right), \left(1, \frac{7}{16}\right), \left(2, \frac{5}{16}\right), \left(3, \frac{1}{16}\right)$ (b) ii. 0.0064 iii. 0.7705

- (c) i. 38 ii. $\frac{1}{19}$ **34.** $\mu = 0.9586, \sigma = 0.0252$ **35.** (a) $\frac{10}{21}$ (b) i. 0.3085 ii. 0.1747 (c) i. 0.2642

- ii. 0.8 **36.** (a) i. 0.8 ii. 0.25 (b) i. 0.4 ii. $E(X) = 0.8, \text{var}(X) = \frac{14}{25}$ **37.** (a) i. $\frac{1}{8}$ ii. $\frac{47}{72}$ iii. $\frac{1}{8}$

iv. $\frac{47}{72}$ v. $\frac{9}{47}$ **38.** (a) i. 0.1353 ii. 0.2707 iii. 0.8647 (b) $\frac{189}{8192} \sim 0.0231$

39. $\frac{43}{60} \approx 0.7167$ **40.** $\frac{117}{145} \approx 0.8069$ **41.** i. $(x, P(X = x))$ values are: $(0, \frac{1}{6}), (1, \frac{1}{3}), (2, \frac{1}{2})$;

$E(X) = \frac{4}{3}, var(X) = \frac{5}{9}$ ii. $\frac{2}{3}$ iii. $\frac{5}{24}$ **42.** (a) 0.0993 (b) \$2.03 per metre. If the lengths are increased, the number of faults per length would also increase, hence, the expected profit/metre would decrease (in this case it would reduce to \$1.21 per metre).

43. i. 0.4 ii. 0.096 iii. 0.225 iv. 0.635 **44.** (a) $\frac{3}{5}$ (b) $(x, P(X = x))$ values are: $(0, \frac{4}{25}),$

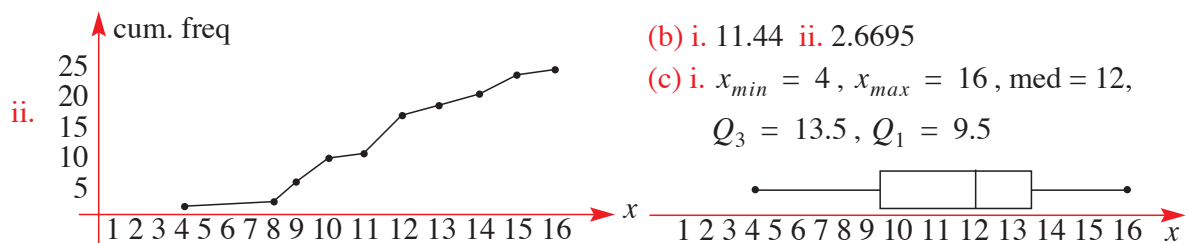
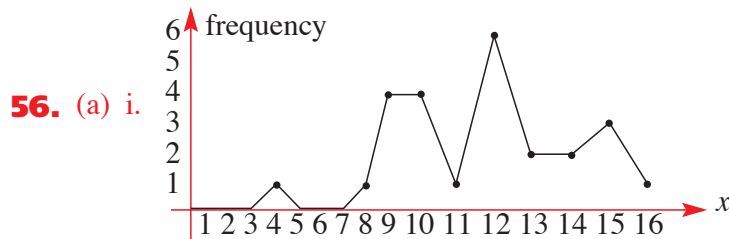
$(1, \frac{12}{25}), (2, \frac{9}{25})$ (c) $E(X) = 1.2, var(X) = 0.48$ (d) $\frac{3}{7}$ **45.** (a) i. $\frac{8}{15}$ ii. $\frac{7}{15}$ iii. $\frac{1}{5}$ iv. $\frac{4}{5}$ v. $\frac{4}{7}$

(b) $\frac{x(p-q) + 100q}{100}$ **46.** $\frac{2}{3}$ **47.** (a) 0.1359 (b) 137.22 (c) $137\frac{1}{3}$ (d) $a = 141.21$ **48.** i. $\frac{2}{3}$

ii. $\frac{2}{9}$ iii. not independent **49.** i. $W = \{2, 3, 4, 5\}$ ii. $\frac{10}{3}$ iii. $\frac{17}{42}$ **50.** i. $b + 6a$ ii. $0 \leq b \leq \frac{1}{3}$

51. i. 4 ii. 0.9084 **52.** i. 0.081 ii. $\frac{4}{13}$ **53.** (a) 0.0169 (b) i. 0.9342 ii. 127 iii. 0.008

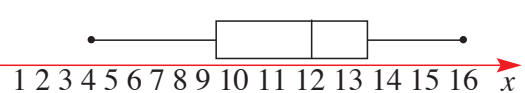
54. i. 0.1587 ii. 0.7745 iii. \$0.23 **55.** (a) i. 0.077 ii. 0.756 iii. 0.167 (b) \$7.61



(b) i. 11.44 ii. 2.6695

(c) i. $x_{min} = 4, x_{max} = 16, med = 12,$

$Q_3 = 13.5, Q_1 = 9.5$



ii. med = 12, mode = 12 iii. 4

57. (a) i. 0.24 ii. 0.36 (b) $172 + 0.96Q$ (c) $Q > 29.17$

58. (a) $P(X \text{ is odd}) = e^{-\lambda}(\lambda + \frac{\lambda^3}{3!} + \frac{\lambda^5}{5!} + \dots)$ (b) $P(X \text{ is even}) = \frac{1}{2}(1 + e^{-2\lambda})$ (c) $\lambda = 1.122$

REVISION EXERCISES – SET D

1. (a) $\frac{x}{\sqrt{x^2 + 4}}$ (b) $2 \cos 2x - 2(2x - 1) \sin 2x$ **2.** (a) 19.8°C (b) 1.6°C per minute (c) 17.3 min

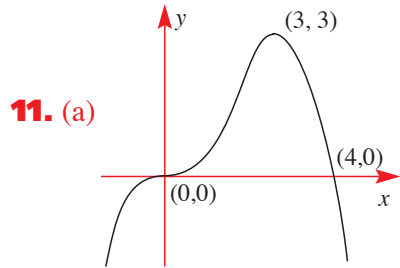
3. (a) $x \in [-1, 0[\cup]0, \infty[$ (b) $x \in]-\infty, 0[\cup [2, \infty[$ **4.** 10 m **5.** (a) $\frac{4x}{(x^2 + 1)^2}$

(b) $-4 \sin 2x \cos 2x$ (or $-2 \sin 4x$) **6.** (a) i. 0 ii. 2 (b) $x \in [-2, 2]$ (c) $x \geq 0$

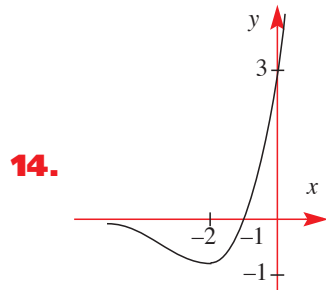
(d) $-\frac{x}{\sqrt{4-x^2}}$, $-2 < x < 2$ **7.** (a) $-\frac{2+h}{(1+h)^2}$, $h \neq 0$ (b) -2 **8.** 1.455 ms^{-1} **9.** (a) Absolute

maximum at $(\pm\frac{1}{\sqrt{2}}, 1)$; local min at $(0, 0)$; x -intercept at $(\pm 1, 0)$ (b) Local min at $(\pm\frac{1}{\sqrt{2}}, 1)$;

asymptotes at $x = \pm 1$, $y = 0$. **10.** i. $6 \cos 2x \sin^2 2x$ ii. $\frac{x+3}{(2x+3)^{3/2}}$



(b) 98° **12.** (b) i. 2 ii. 72 cm^3 **13.** i. $-\frac{3x}{\sqrt{1-3x^2}}$ ii. $\frac{e^x}{(1+e^x)^2}$



15. (a) $h = \frac{1000}{\pi r^2}$ (b) radius = 5 cm, height = 12.7 cm

16. (a) $3x^2h + 3xh^2 + h^3$ (b) $3x^2 + 3xh + h^2$ (c) $3x^2$ **17.** (a) $p'(t) = 0.8(1 - 0.02t)e^{-0.02t}$
(b) ~ 38.3 million (c) i. decreasing ii. ~ 0.1 million/year (d) 50 years time, i.e., 2030; 42.2 million

18. 76222 cm^3 **19.** i. $\frac{1 + \cos x + x \sin x}{(1 + \cos x)^2}$ ii. $\frac{x}{x^2 + 1}$ **20.** (a) $12 + 6h + h^2$, $h \neq 0$ (b) 12

21. (a) i. 283 sec ii. 250 sec (c) 244 sec **22.** $a = -1$, $b = 6$, $c = -9$ **23.** (a) $\frac{4\sqrt{3}}{3}$

(b) i. $-6 \sin 3x \cos 3x$ ii. $\frac{x}{2} + \frac{1}{12} \sin 6x + c$ **24.** $y = x - 2$ **25.** max = 64, min = $-\frac{2187}{256}$

26. i. $-\frac{3 \cos x}{2\sqrt{2-3 \sin x}}$ ii. $(2 \cos \frac{1}{2}x - \frac{1}{2} \sin 2x)e^{2x}$ iii. $\frac{1 - \ln x}{x^2}$, $x > 0$ **27.** (a) $V = \pi r^2 h + \frac{4}{3} \pi r^3$

(b) $P = 2\pi k r h + 6\pi k r^2$ (c) $P = \frac{2kV}{r} + \frac{10\pi}{3} k r^2$ (d) $0 < r < (\frac{3V}{4\pi})^{1/3}$ (e) $r = (\frac{3V}{10\pi})^{1/3}$

28. (b) $[\frac{a}{4}, \frac{a}{2}]$ (c) $\frac{\sqrt{3}}{36} a^3$ cubic units **30.** i. $-6 \sin 2x \cos^2 2x$ ii. $\frac{1-2x^2}{\sqrt{1-x^2}}$ **31.** $y = -\frac{1}{2}x + 2$

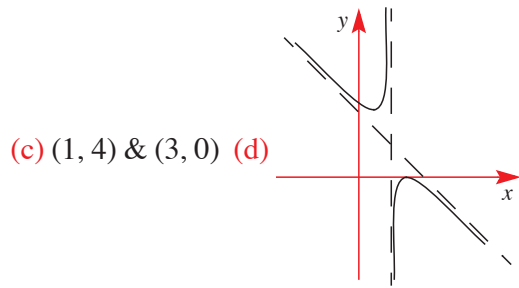
32. 0.5 cm s^{-1} **33.** (a) $[0, 5]$ (b) use g.c (c) 0.625 (d) $a = \frac{1}{2} - \frac{1}{5}t$, $0 \leq t \leq 5$

34. (c) Minimum, $3\pi a^2 (\frac{5}{3})^{1/3}$; Maximum $3\pi a^2 (\frac{9}{4})^{1/3}$ **35.** (a) $-e^{-x}(\cos x + \sin x)$ (b) 1

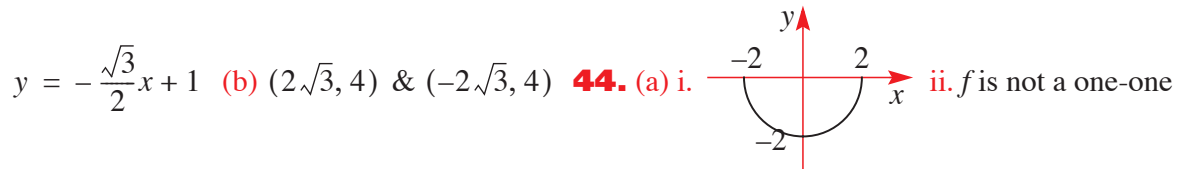
36. i. $V = \frac{4}{27} \pi h^3$ ii. 64 sec iii. falling at 1.15 cm min^{-1} **37.** (a) $x < \frac{1}{2}$ or $x > \frac{1}{2}$

(b) $-\frac{1}{2} < x < 0$ or $0 < x < \frac{1}{2}$ **38.** $\frac{4}{3}$ **39.** (b) i. $AB = 200 \tan \theta$ ii. $20 \sec^2 \theta$ iii. Point A iv. yes

40. (a) $\sin^{-1}(x) + \frac{x}{\sqrt{1-x^2}}$ (b) $y' = \frac{2x^2 - \ln(x^2 + 1)}{x^2(x^2 + 1)}$ **41.** (a) $(0, \frac{9}{2})$ (b) $y = 4 - x$ and $x = 2$

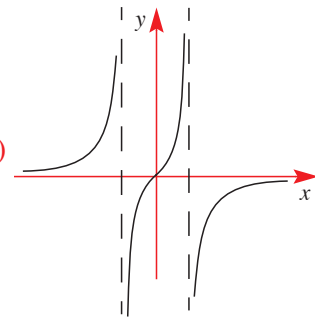


42. (a) $\frac{12 - 8x^2}{(2x^2 + 3)^2}$ (b) $\frac{8}{9}\sqrt{3}$ **43.** (a) $y = \frac{\sqrt{3}}{2}x + 1$,



function (b) $c = -2$ (c) i. $f \circ g(x) = -2|\sin x|$, range = $[-2, 0]$, domain = \mathbb{R} ii. $\mathbb{R} - \{n\pi, n \in \mathbb{Z}\}$

45. 0.16 rad per sec 46. $y = e^{-x}$ **47.** (a) $x = 1, x = -1, y = 0$ (c)



48. (a) $(4 \sin 2x - 3 \cos 2x) \cdot e^{-x}$ (b) $\frac{2 - 3x^2}{x^2 \sqrt{(1-x^2)^3}} + \frac{2 \cos^{-1} x}{x^2}$

49. (a) $y = (3 \ln 3)x + 3 - 3 \ln 3$ (b) $y = \frac{1}{\ln 2}(x + 1)$

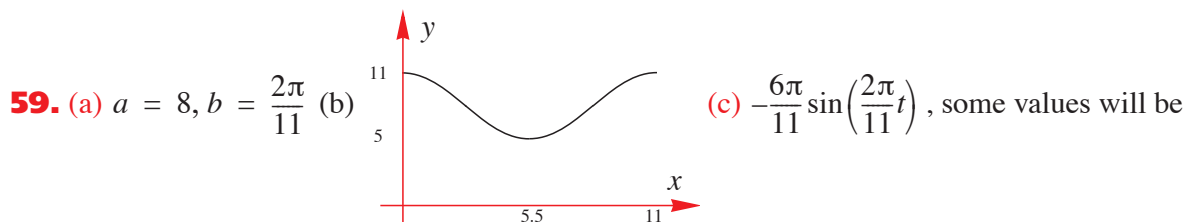
50. dom = $\{x | x < -4\} \cup \{x | -4 < x < 0\} \cup \{x | x > 0\}$ **51.** $\sim 2.77 \text{ km h}^{-1}$ **52.** $\frac{1}{25\pi} \text{ ms}^{-1}$

53. 2.83 ms^{-1} **54.** (a) 27 sec (b) 746 m **55.** (a) $A = 318 - 6x - \frac{1176}{x^2}, 4 \leq x \leq 49$

(b) $x = 14, y = 21$ **56.** (a) $a = \frac{4}{3}, b = -1, c = 4$ **57.** (a) i. $0 \leq x \leq \frac{1}{3}L$ (b) i. $x = \frac{3L}{\sqrt{3\pi + 9}}$

ii. local min. iii. use a graphics. calc. (c) i. $x = \frac{3L}{\sqrt{3\pi + 9}}$ ii. $x = 0$ **58.** (a) i. $\frac{2x}{(x^2 + 1)^2}$

ii. $-\frac{66}{(3x + 2)^3}$ (b) $(20 \log_e 5)y = -x + 1 + 400 \log_e 5$ (c) $\frac{1}{\ln 2}$ (d) $1.5 \ln 2 \text{ ms}^{-1}$



negative indicating that the tide is going out. (d) $-8 \sin\left(\frac{2\pi}{11}t\right)$

60. (b) $k = 250$ (c) $I = 250 \sin \theta [2 \cos^2 \theta - \sin^2 \theta]$ (d) 212 m (e) 96.23 (f) ii. ~ -0.68

61. 0.032 rad per sec

REVISION EXERCISES - SET E

1. (a) i. $\frac{91}{3} + \ln 4$ ii. $\frac{4}{15}(1 + \sqrt{2})$ (b) i. $-2\sqrt{1-x^2} + c$ ii. $\frac{1}{3} \tan^{-1}\left(\frac{4x}{3}\right) + c$ iii. $\frac{1}{2} \sin^{-1}(2x) + c$

2. (a) $N = ke^t + 2, N > 2$ (b) 3 m (c) $y = \tan\left(t - \frac{\pi}{4}\right)$ **3.** $v = 2 - \sqrt{t+1}, t \geq 0$

4. (a) $-7\mathbf{i} + 6\mathbf{j} + \mathbf{k}$ (b) -8 (c) $a = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$ **5.** (a) $\frac{1}{48}$ (b) $\frac{\pi}{3}$ (c) 2

6. (b) i. $\pi \int_0^{0.5} 4y^2 dy + \pi \int_{0.5}^1 \left(\frac{1}{y} - 1\right) dy$ ii. 1.13 cubic units **7.** 327 cm **8.** $\frac{dm}{dt} = -\frac{7m}{100+3t}$

9. (a) $4x - y - 4z = -2$ (b) $-x + 7y + 19z = 42$ (c) (10, 10, 3) **10.** (a) $\frac{\pi}{3}, \frac{4\pi}{3}$

(b) $\left\{x \mid \frac{\pi}{3} < x < \frac{4\pi}{3}\right\}$ (c) 4 sq. units **11.** 720 m³ **12.** (a) i. $x = \frac{7}{2} + \lambda, y = \lambda, z = \frac{9}{2} + \lambda, \lambda \in \mathbb{R}$

ii. $\frac{x-3.5}{1} = \frac{y}{1} = \frac{z-4.5}{5}$ (b) i. (-5, 3, 2) ii. (8, 11, 0) (c) $2x + 5y - z = -8$ **13.** $a = 1, b = 6$

14. 0 **15.** (a) $-8\mathbf{i} + 11\mathbf{j} + 9\mathbf{k}$ (b) $a = -\frac{2}{5}, b = \frac{4}{5}$ (c) $5\sqrt{3}$ sq. units

16. (a) Area = $A = \frac{8}{15}h^{3/2}$, Volume = $V = 0.48h^{3/2}$ (b) $\frac{5}{144}$ m/min **17.** (a) (-1, 4), (1, -2),

(3, 0) (b) use g.c (c) $\frac{16}{3} - 3 \log_e 3$ sq. units **18.** (a) $2 - \frac{3}{4}\sqrt{3}$ (b) $\log_e 3$ **19.** (a) $A \equiv (2, 2e^{-1})$

(b) i. $y = x$ ii. $\frac{d}{dx}(xe^{-x/k}) = \left(1 - \frac{x}{2}\right)e^{-x/2}$ iii. $4 - 2(2+a)e^{-a/2} - \frac{1}{2}a^2$ (c) i. $(2x - x^2)e^{-x}$

ii. $\pi(2 - 10e^{-2})$ cubic units **20.** (a) $f(x) = \frac{1}{5}(2x-1)^{5/2} + \frac{1}{3}(2x-1)^{3/2} + c$ (b) i. $\frac{\pi}{12}, \frac{\pi}{48}$

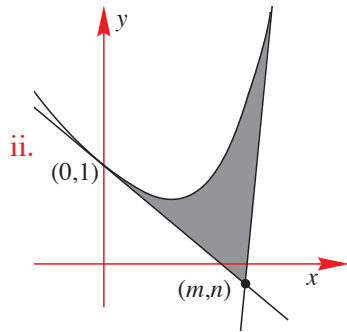
ii. $\frac{1}{9}$ (c) i. use g.c. ii. $\frac{\pi}{3}$ sq. units iii. $\frac{\pi}{2} \ln 3$ cubic. units **21.** $k = 1$ or 3 **22.** $t = 2, (16, -8, 4)$

23. (a) i. $x = -\frac{3}{k-3}, y = \frac{8-k}{k-3}, z = \frac{k-5}{k-3}$ ii. $k \neq -2, 3$ (b) $k = 3$ (c) i. $k = -2$ ii. $x = 2 - \lambda,$

$y = -2, z = \lambda, \lambda \in \mathbb{R}$ **24.** $A^2 = \mathbf{I}_{3 \times 3}; (-1, -3, 4)$ **25.** $BX = \frac{1}{2}A, X = \frac{1}{18}BA,$

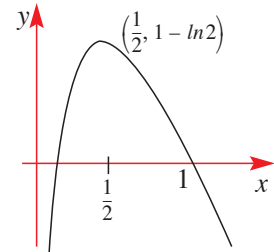
$AX = \frac{1}{18}ABA$ **26.** (a) $A \equiv \left(\frac{1}{2} \ln 2, 2(1 - \ln 2)\right)$ (b) $\frac{1}{2}(e^2 - 5)$ sq. units

(c) i. At (0,1): $y = -2x + 1$ At (1, $e^2 - 4$): $y = (2e^2 - 4)x - e^2$



iii. $\frac{1}{2}(e^2 - 5)$ sq. units (d) ii. $\frac{\pi}{12}(3e^4 - 24e^2 + 37)$ cubic units

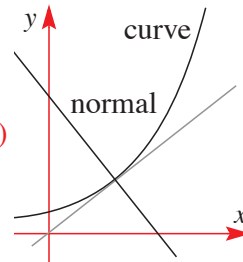
27. (b) i. $0 < x < 0.5$ ii. $x = 0.5$ iii. $x < 0$ or $x > 0.5$ (c) $(\frac{1}{2}, 1 - \ln 2)$ (d)



(e) i. $y = -x + 1$ ii. $y = x - 1$ (f) i. $\frac{3}{8} - \frac{1}{2} \ln 2$ sq. units ii. $\frac{1}{8} + \frac{1}{2} \ln 2$ sq. units 28. $A = 0, B = 0.5$

29. (a) $\frac{7}{12}$ sq. u (b) $\frac{7}{15}\pi$ cubic units 30. (a) $a = 2; f^{-1}(x) = 2 + \sqrt{x}, x \geq 0$

(b) $\frac{136}{3}\pi$ cubic units 31. (a) $y = -ex + e + e^{-1}$ (b)



(c) $\frac{1}{2}e + e^{-2}$ sq. units

32. (a) $\frac{4}{3}$ sq. units (b) $\frac{64}{15}\pi$ cubic units 33. (a) $x = \frac{1}{3}[20 - 14e^{-0.1t}]$ (b) 19 minutes

34. $\frac{\tan^{-1}\sqrt{x}}{2\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{1+x}, 2\sqrt{3}\pi - \frac{\pi}{2} - \ln 2$ 35. (a) $\frac{dh}{dt} = \frac{V - k\sqrt{h}}{\pi h^2}$ (b) $(\frac{V}{k})^2$ m

36. (a) $3i - j - 2k$ (b) 100° (c) $4i - 3j - 3k$ 37. (b) $A^{-1} = \frac{1}{3}(A - 2I)$

38. (a) $1 - \frac{x}{\sqrt{1-x^2}} \cdot \sin^{-1}x, \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ (b) i. $\frac{2}{5}(1-x)^{5/2} - \frac{2}{3}(1-x)^{3/2} + c$ ii. $e^x - \ln(e^x + 1) + c$

(c) $\frac{1}{3} \ln 2$ 39. $\frac{64}{15}\pi$ cubic units 40. (a) $2\sqrt{2}$ (b) $t = 5, b = 0.4$

41. (a) $\frac{dx}{dt} = \frac{50-x}{10}, t = 0, x = 6$ (b) 0.32 kg per litre 42. (a) $\frac{1}{a-2} \begin{bmatrix} -3 \\ 2a-1 \end{bmatrix}$ (b) $a = 2$

(c) $x = -3, y = 5$ 43. (a) $a = 1$ (b) 1.25 (c) $\frac{13}{48}$ (d) $\frac{19}{64}$ 44. (a) i. 90° ii. $\frac{7}{2}\sqrt{26}$ unit²

(b) i. $s + 3p$ ii. $s + 2p$ iii. $\frac{1}{2}s + 2p$ iv. $-\frac{1}{2}s + 2p$ 45. (a) 1 (b) $A^n = \begin{bmatrix} 2^n & (2^n - 1)a \\ 0 & 1 \end{bmatrix}$

(c) $a = -\frac{2}{9}$ 46. (a) 27° (b) $\frac{1}{2}\sqrt{17}$ unit² 47. (a) i. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 7 \end{pmatrix}, \lambda \in \mathbb{R}$

ii. $x = 2 + 3\lambda, y = -3 + 7\lambda, \lambda \in \mathbb{R}$ iii. $\frac{x-2}{3} = \frac{y+3}{7}$ (b) $-i + 11j$ (c) i. no ii. lines are skew

48. (a) i. $2x + 3y - 4z = 2$ ii. 90° (b) i. $28^\circ 35'$ ii. $32^\circ 25'$ **49.** $(-8, 11, -6)$ **50.** (a) $a = \frac{3}{2}$

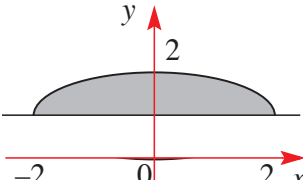
(b) $b = \frac{3}{2}, c = \frac{1}{3}$ **51.** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -19 \\ -5 \end{pmatrix}$ **52.** (a) $\hat{c} = \frac{1}{\sqrt{77}}(-5i + 4j + 6k)$

(c) $2x + y + 3z = 4$ **53.** $k = -1, 1$ or 2 **54.** (a) i. $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ ii. $\frac{1}{2} \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$

(b) $x + y - z = 1, 3x + 4y + 6z = 7$ (c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 10 \\ -9 \\ 1 \end{pmatrix}$ **55.** (a) i. $\frac{\pi}{2}$ ii. $2\pi - 4$

(b) $\frac{\pi^2}{2} + \pi$ cubic units (c) i. $y = -x + 2$ ii. $\frac{3}{2} - e^{-1}$ sq. units **56.** (a) see page 362

(b) $\frac{\pi\sqrt{3}}{3} + \ln\frac{1}{2}$ sq. units **57.** (b) i. $a = \sqrt{3}$ ii. $b = \frac{1}{6}$ **58.** $\frac{\pi}{2}(c^2 - e^{-2})$ cubic units

59. (a)  (b) $2\pi^2 + \frac{8\pi}{3}$ cubic units **60.** (a) 5 (b) $\frac{5}{3}\sqrt{5}$ **61.** (b) 76 sec

62. $I_1 \sqrt{\frac{I_1}{I_2}}$ **63.** (a) i. $a = 3, b = -2$ ii. $a = \frac{1}{3}(2\lambda - 1), b = \lambda, \lambda \in \mathbb{R}$ (b) $(-12, -10, -2)$

(c) i. $\frac{1}{2}\sqrt{629}$ ii. $x - 12y + 22z = 0$ iii. $\mathbf{r} = \lambda \begin{pmatrix} 1 \\ -12 \\ 22 \end{pmatrix}$ **64.** (a) i. $\frac{1}{2}(x^2 + 1)\arctan x - \frac{1}{2}x + c$

ii. $x \cdot \arcsin x + \sqrt{1 - x^2} + c$ (b) i. $\frac{1}{4}[(1 - 2a^2)\cos 2a + 2\sin 2a - 1]$ ii. $\frac{5}{27}e^3 - \frac{2}{27}$

65. (a) $t = 1$ & $t = 5$. ii. use a g.c. (b) $t = 2.21$ **66.** 40.5°C

Note: A Solutions Manual – which includes fully worked solutions to most questions in the textbook is also available. Check the ibid press website for updates and availability.

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